Discounting and relative consumption

Olof Johansson-Stenman, Thomas Sterner [*,1]

Department of Economics, School of Business, Economics and Law, University of Gothenburg, Box 640, S-40530 Gothenburg, Sweden

ARTICLE INFO

ABSTRACT

We analyze optimal social discount rates when people derive utility from relative consumption, i.e. their own consumption level relative to the consumption level of others. We compare the *social*, *private*, and conventional *Ramsey* rates. Assuming a positive growth rate, we find that (1) the social discount rate exceeds the private discount rate if the importance of relative consumption increases with consumption, and that (2) the social discount rate is lower than the Ramsey rate given quasi-concavity in own and others' consumption and risk aversion with respect to others' consumption. Numerical calculations demonstrate that the latter difference may be substantial and have important implications for long run environmental issues such as global warming.

© 2015 Elsevier Inc. All rights reserved.

Introduction

The theory and practice of discounting is central to economics (e.g., Arrow and Lind, 1970; Arrow et al., 1996; Frederick et al., 2002) and essential for dealing with very long-term phenomena. With the increased attention to environmental issues such as climate change, interest in discounting has experienced a revival (see Gollier, 2010; Gollier and Weitzman, 2010; Weitzman, 2010; Arrow et al., 2013 for recent contributions). For example, essentially the entire economics debate in the wake of the *Stern Review* (Stern, 2006) largely focused on the discount rate used—not on climate science or the assessment of costs and benefits of mitigation, for which there are still very large uncertainties (see, e.g., Dasgupta 2007, 2008; Dietz and Stern, 2007; Nordhaus, 2007a, 2007b; Weitzman, 2007a, 2007b; Brekke and Johansson-Stenman, 2008; Heal, 2008; Sterner and Persson, 2008; Cameron et al., 2009; Howarth, 2009; Karp and Tsur, 2011). The primary reason is, of course, that most of the consequences of climate change will occur far into the future and thus the discount rate has a dramatic effect on their present value.

This paper is, as far as we know, the first to incorporate relative consumption effects into the theory of social discounting. Yet, the idea that humans value consumption in a social context and in relation to others' consumption is far from new. In fact, classical economists—such as Adam Smith, John Stuart Mill, and Alfred Marshall—emphasized such concepts, and modern research on the subject dates back at least to Duesenberry (1949). There is now a substantial body of empirical evidence suggesting that people not only derive utility from their absolute consumption but are also concerned with their own consumption relative to that of others.[2]

---

* Corresponding author.
  E-mail addresses: Olof.Johansson@economics.gu.se (O. Johansson-Stenman), Thomas.Sterner@economics.gu.se (T. Sterner).
[1] Fax: +46 31 786 10 43.

[2] This includes happiness research (e.g., Blanchflower and Oswald, 2004; Ferrer-i-Carbonell, 2005; Luttmer, 2005), questionnaire-based experiments (e.g., Johansson-Stenman et al., 2002; Solnick and Hemenway, 2005; Carlsson et al., 2007), and brain science (e.g., Fliessbach et al., 2007). There are also

There is also a growing literature dealing with various kinds of optimal policy responses to such relative consumption effects.[3] For example, Aronsson and Johansson-Stenman (2008, 2010) show, in a static and a dynamic model, respectively, that optimal marginal income taxes are likely to be substantially larger under relative consumption effects than in a conventional case, i.e., where people only care about their absolute consumption level.

Arrow and Dasgupta's paper (2009) is closest to ours, as it explicitly deals with the implications of relative consumption effects for intertemporal resource allocation. They show that concern for relative consumption does not necessarily lead people to consume more today than is socially optimal, since they are also concerned with relative consumption (and hence produce positional externalities) in the future; this is also shown in slightly different settings by Wendner (2010a, 2011). Aronsson and Johansson-Stenman (2014a) derive optimal provision rules for state-variable public goods (interpreted as the global climate) over time under relative consumption concerns. However, the issue of how discount rates are affected by relative consumption effects has not been analyzed before.

We will in most of the paper build on a general utility formulation, meaning that our results are not due to a particular functional form. Still, we will keep the model as simple as possible allowing us to focus on the implications of relative consumption concerns, and for example abstract from within-generation inequality, population growth rates and various uncertainties. In the section "Private and social discount rates", we analyze the question of whether and, if so, how the *social* discount rate (when positional externalities are taken into account) is lower or higher than the *private* discount rate (when people do not take into account that their consumption affects others—although they still take into account that they themselves are affected by others' consumption). We show that an important condition derived by Arrow and Dasgupta (2009) for when private and social consumption paths coincide over time translates to the condition for when the private and social discount rates are the same. We then explore the conditions when the social discount rate exceeds the private one, and vice versa. We express these conditions in terms of the *degree of positionality*, a measure reflecting the extent to which relative consumption matters. For a positive growth rate, we show that if the degree of positionality increases with the consumption levels, consistent with some empirical evidence, then the social discount rate exceeds the private one. We also show that this discrepancy can be internalized by time-varying consumption taxes.

In the section "Comparisons with the conventional Ramsey discounting rule", we continue by analyzing a related but distinct issue—relevant from a climate policy perspective—namely whether, and if so how, the conventional optimal social discounting rule, the so-called Ramsey discounting rule (Ramsey, 1928), should be modified in the presence of relative consumption effects.[4] The conventional Ramsey discounting rule says that the optimal discount rate equals the pure rate of time preference plus the product of the individual degree of relative risk aversion multiplied by the growth rate. Hence, this formulation does not take into account that others' consumption will also change in the future.

The formula describing the optimal social discount rate in the presence of relative consumption effects can be written in a form similar to the Ramsey formula. The only difference is that the individual degree of relative risk aversion is replaced by what we denote here as the *social* degree of relative risk aversion. By this we mean a corresponding measure of risk aversion had the individual made a risky choice on behalf of his or her whole generation. Assuming a positive consumption growth rate, we show that the social discount rate is lower than the Ramsey discount rate, if preferences are quasi-concave in own and reference consumption (consisting of others' average consumption) and concave in reference consumption. The latter means that individuals prefer that others have a certain consumption level compared to the case where others' consumption is uncertain with the same expected value. We show moreover that the social discount rate is higher than the private rate if the degree of positionality increases with consumption. Taken together, this implies that the social discount rate is *higher* than the private rate but *lower* than the Ramsey rate, if the degree of positionality increases with consumption and preferences reflect risk aversion with respect to reference consumption and are quasi-concave with respect to own and reference consumption. It is worth emphasizing that we throughout the paper assume identical individuals who in equilibrium consume the same amount in each time period. This implies that the identified effects of relative consumption concerns for the optimal discount rate do not at all depend on assumptions of inequality, or aversion to inequality.

Finally, we illustrate quantitatively in the section "Numerical illustration and orders of magnitude" how the overall long-term social discount rates suggested by Stern (2006) and Weitzman (2007b) would be modified when taking relative consumption effects into account. We conclude that these modifications may be substantial with commonly used functional forms and reasonable parameter values, and that they are potentially very important for the economics of climate change. The "Conclusion and discussion" offers some final remarks and observations.

(*footnote continued*)
recent evolutionary models consistent with relative consumption concerns (Samuelson, 2004; Rayo and Becker, 2007). Stevenson and Wolfers (2008) constitute an exception in the happiness literature, claiming that the role of relative income is overstated.

[3] See, e.g., Boskin and Sheshinski (1978), Layard (1980), Oswald (1983), Ng (1987), Brekke and Howarth (2002), Abel (2005), and Wendner and Goulder (2008). Clark et al. (2008) provide a good overview of both the empirical evidence and economic implications of relative consumption concerns.

[4] Other papers have dealt with the issue of whether, and if so how, the externalities induced by relative consumption effects affect the consumption pattern over time; see e.g. Wendner (2010b) and references therein.

Private and social discount rates

In this section, we analyze and compare the private and social discount rates between two arbitrary points in time, when people care about relative consumption. At the end of this section, we also determine the corresponding internalizing consumption taxes.

Preferences and objectives

Following Arrow and Dasgupta (2009), society consists of identical individuals, with a population size normalized to one, who, at time t, consume c_t in equilibrium and who, in addition to absolute consumption, also care about relative consumption. The latter depends on own consumption as well as the reference consumption z_t (the average of others' consumption), such that $R_t = r(c_t, z_t)$. The individual instantaneous utility (or felicity) at time t is:

$$U_t = u(c_t, R_t) = u(c_t, r(c_t, z_t)) = v(c_t, z_t), \tag{1}$$

where $u_{c_t} > 0$, $u_{R_t} > 0$, $v_{c_t} > 0$, $v_{z_t} < 0$, $u_{c_t c_t} < 0$, $v_{c_t c_t} < 0$, $r_{c_t} > 0$, $r_{z_t} < 0$; where sub-indices reflect partial derivatives, i.e., $u_{c_t} \equiv \partial u(c_t, R_t)/\partial c_t$, etc., and where u, v, and r are twice continuously differentiable.

Both the u and v formulations in Eq. (1) are used extensively in this paper, since they allow us to vary which variable is constant, i.e., either relative consumption R_t or others' consumption z_t. Throughout the paper, we assume the same uniform and normalized (to one) population in each period. In doing this we completely abstract from intra-generational distribution issues. An advantage with this assumption, in addition to keeping the model tractable and transparent, is that it makes clear that the importance of relative consumption concerns for discounting does not depend on assumptions regarding inequality or inequality aversion.

Moreover, we let r satisfy the criterion that it is unaffected if own consumption and others' consumption are changed equally, i.e., $r_{c_t} = -r_{z_t}$. This assumption is fairly innocuous as long as individuals are identical within each generation. For example, it encompasses the most commonly used comparison forms, i.e., the difference comparison form where $R_t = c_t - z_t$ (e.g., Akerlof, 1997; Ljungqvist and Uhlig, 2000; Bowles and Park, 2005; Carlsson et al., 2007), the ratio comparison form where $R_t = c_t/z_t$, (e.g., Boskin and Sheshinski, 1978; Layard, 1980; Wendner and Goulder, 2008), and the more flexible functional form suggested by Dupor and Liu (2003), which includes both the difference and the ratio forms as special cases, as long as $z_t = c_t$ (see below).

In addition, we make the common assumption of (weak) keeping-up-with-the-Joneses (KUJ) property, such that $v_{c_t z_t} \geq 0$. This assumption implies that people want to consume more (less) in those time periods when others consume more (less).[5] Dupor and Liu (2003) use a corresponding definition of KUJ preferences in a static model where utility also depends on leisure (in addition to absolute and relative consumption). In such a model, the consumption adjustment is thus due to a modification of the consumption-leisure tradeoff, rather than due to an adjusted consumption path over time. KUJ preferences are then naturally defined as preferences where the marginal rate of substitution between consumption and leisure increases with others' consumption, as this implies that the individual's consumption increases (and leisure decreases) when others' consumption increases. Thus, both the present definition and the one by Dupor and Liu (2003) imply that people want to consume more when others consume more, and vice versa. See also Gali (1994), Carroll et al. (1997) and Wendner (2010a) for other treatments of KUJ preferences in the literature. Finally, we also assume perfect forecast, such that both individuals and the government know future consumption of all individuals with certainty. Other properties of these utility functions will be discussed in subsequent sections.

Each individual maximizes the flow of instantaneous utility over time, subject to a fixed pure rate of time preference—or the utility discount rate δ—implying the following maximization problem:

$$\max_{c_\tau} w^p \equiv \int_0^T u(c_\tau, r(c_\tau, z_\tau)) \exp(-\delta\tau) d\tau = \int_0^T v(c_\tau, z_\tau) \exp(-\delta\tau) d\tau. \tag{2}$$

Hence, the individual takes others' consumption z_t as given at each moment in time, implying that the individual takes into account that a changed consumption path also implies a changed path of relative consumption. While in equilibrium $z_t = c_t$ at each moment in time, the individual will nevertheless take z_t as given, which resembles the conventional externality assumption in the representative consumer model.

The social planner, who is welfarist and respects individual preferences, in contrast maximizes the flow of instantaneous utility for *all* individuals (and hence also takes into account the externalities through relative consumption). As a consequence, since all individuals are identical and consume the same amount in equilibrium, the social planner takes R_t

---

[5] It is straightforward to show that in a model where an individual maximizes the present value of instantaneous utility (that depends on both own consumption and an external measure of reference consumption) subject to a conventional intertemporal budget restriction, then it follows that an exogenous increase in the reference consumption in one specific time period increases the individual's consumption in that time period if and only if $v_{c_t z_t} > 0$ (the derivation is available from the authors upon request). In contrast, when $v_{c_t z_t} < 0$, and if the magnitude of this effect is sufficiently large, it may be both privately and socially optimal for identical individuals to have different consumption patterns over time, such that an individual will consume more when others consume less and vice versa. This corresponds to what Dupor and Liu (2003) refer to as Running-away-from-the-Joneses preferences.

as given. Thus, the social planner solves the following maximization problem:

$$\max_{c_\tau} w^s \equiv \int_0^T u(c_\tau, r(c_\tau, c_\tau)) \exp(-\delta\tau) d\tau = \int_0^T v(c_\tau, c_\tau) \exp(-\delta\tau) d\tau. \tag{3}$$

Private and social discount rates

Consider now a small project undertaken and paid for in terms of reduced consumption at time zero that results in increased consumption at time t. Both the government and the individuals will thus take the consumption levels in all other time periods as fixed, which is assumed throughout the paper. Note that we are only concerned with given consumption paths and since we deal with small projects as is common in the discounting literature,[6] our results are independent of any production assumptions, including issues of technical change. If the individual is indifferent between undertaking such a project and not, the private discount rate per time unit between time zero and t, given by ρ^p, is implicitly defined by:

$$\frac{\partial w^p / \partial c_t}{\partial w^p / \partial c_0} = \exp(-\rho^p t) .$$

Note that this is true for any given consumption path, whether optimal or not. Solving for ρ^p gives[7]

$$\rho^p(t) = -\frac{1}{t} \ln \frac{\partial w^p / \partial c_t}{\partial w^p / \partial c_0}. \tag{4}$$

Similarly, if the social planner is indifferent between undertaking such a project and not, the social discount rate per time unit between time zero and t, given by ρ^s, is implicitly defined by $\partial w^s / \partial c_0 = \exp(\rho^s t) \partial w^s / \partial c_t$, implying that:

$$\rho^s(t) = -\frac{1}{t} \ln \frac{\partial w^s / \partial c_t}{\partial w^s / \partial c_0}. \tag{5}$$

Next, from equation (2) we have

$$\frac{\partial w^p / \partial c_t}{\partial w^p / \partial c_0} = \frac{v_{c_t}}{v_{c_0}} \exp(-\delta t),$$

which substituted into Eq. (4) implies:

$$\rho^p(t) = \delta - \frac{1}{t} \ln \frac{v_{c_t}}{v_{c_0}}. \tag{6}$$

Similarly, we have from equation (3) that

$$\frac{\partial w^s / \partial c_t}{\partial w^s / \partial c_0} = \frac{v_{c_t} + v_{z_t}}{v_{c_0} + v_{z_0}} \exp(-\delta t),$$

which substituted into equation (5) implies:

$$\rho^s(t) = \delta - \frac{1}{t} \ln \frac{v_{c_t} + v_{z_t}}{v_{c_0} + v_{z_0}}. \tag{7}$$

Combining Eqs. (6) and (7) immediately implies

$$\rho^s(t) - \rho^p(t) = \frac{1}{t} \ln \frac{1 + v_{z_0}/v_{c_0}}{1 + v_{z_t}/v_{c_t}},$$

and hence the following result:

Proposition 1. $\rho^s(t) = \rho^p(t)$ $\forall t$ *if and only if* $v_{z_t} = \beta v_{c_t}$ $\forall t$, *where* β *is a constant.*

Proof. The "if" part follows directly since $v_{z_t} = \beta v_{c_t}$ $\forall t$ implies that $\rho^s(t) - \rho^p(t) = (1/t)\ln(1+\beta/1+\beta) = 0$ $\forall t > 0$ (and we only consider positive t since we solely deal with discount rates of the future). Consider next the "only if" part. Suppose there exists a t for which $v_{z_t} = \beta v_{c_t}$ does not hold and where instead $v_{z_t} = \delta v_{c_t}$, where $\delta \neq \beta$. The discount rate (between 0 and t) difference is then given by $\rho^s(t) - \rho^p(t) = (1/t)\ln(1+\beta/1+\delta) \neq 0$. Hence, a necessary condition for $\rho^s(t) = \rho^p(t)$ $\forall t$ is that $v_{z_t} = \beta v_{c_t}$ $\forall t$.

Proposition 1 says that the social and private discount rates between all time periods and the present are equal if and only if the ratio between the marginal disutility of others' consumption and the marginal utility of own consumption is

---

[6] See, e.g., Gollier (2012) for a comprehensive state-of-the-art overview of social discounting.
[7] The reason we write $\rho^p(t)$ (instead of ρ_t^p) is that we here consider the discount rate over the discrete time interval from time zero to time t. In the next section, we will use the notation ρ_t^p for the instantaneous discount rate at time t.

constant for all time periods.[8] As such, it resembles Proposition 1 in Arrow and Dasgupta (2009), which says—based on a similar but slightly different model—that the privately and socially optimal consumption paths coincide, if and only if, for all t, $v_{z_t} = \beta v_{c_t}$.

In order to explore the differences between the social and private discount rates in a way that allows for a straightforward economic interpretation, we introduce a measure (following, e.g., Johansson-Stenman et al., 2002; Aronsson and Johansson-Stenman, 2008) that reflects the extent to which relative consumption matters.

Definition 1. The degree of positionality is defined by

$$\gamma_t \equiv \frac{u_{R_t} r_{c_t}}{u_{c_t} + u_{R_t} r_{c_t}}. \tag{8}$$

Thus, $\gamma_t \in (0, 1)$ reflects the fraction of the overall utility increase from the last dollar consumed that is due to increased *relative* consumption. As γ_t approaches zero, *relative* consumption at time t does not matter at all at the margin, taking us back to the standard model. In the other extreme case, where γ_t approaches one, *absolute* consumption does not matter at all (i.e., all that matters is relative consumption).

From Eq. (1), we have that $v_{z_t} = u_{R_t} r_{z_t}$ and $v_{c_t} = u_{c_t} + u_{R_t} r_{c_t}$, together implying that:

$$-\frac{v_{z_t}}{v_{c_t}} = -\frac{r_{z_t}}{r_{c_t}} \gamma_t = \gamma_t. \tag{9}$$

Substituting Eq. (9) into Eq. (7) gives, after some straightforward manipulations,

$$\rho^s(t) = \delta - \frac{1}{t} \ln\left(\frac{v_{c_t}}{v_{c_0}} - \frac{v_{c_t}}{v_{c_0} + v_{z_0}}(\gamma_t - \gamma_0)\right) = \delta - \frac{1}{t} \ln\left(\frac{v_{c_t}}{v_{c_0}} \frac{1 - \gamma_t}{1 - \gamma_0}\right), \tag{10}$$

implying that the social discount rate increases in the positionality difference $(\gamma_t - \gamma_0)$. By combining Eqs. (6) and (10) we get

$$\rho^s(t) - \rho^p(t) = \frac{1}{t} \ln\frac{1 - \gamma_0}{1 - \gamma_t},$$

which immediately gives us the following result:

Proposition 2. $\rho^s(t) > (<) \, \rho^p(t)$, if and only if $\gamma_t > (<) \, \gamma_0$.

Consider the reasonable case with a positive growth rate, such that the consumption level is larger at time t than at time 0. Then Proposition 2 says that if relative consumption becomes more important (compared to absolute consumption) when consumption increases, then the social discount rate is higher than the private one. The intuition is straightforward: when relative consumption becomes more important for higher consumption levels, a larger share of overall consumption increases is "waste" in terms of positional externalities; and since consumption increases over time, this waste share increases over time too. This implies that future consumption becomes less valuable, compared to the present one, from a social point of view, and compared to the case where this waste is not taken into account. Consequently, the social discount rate is higher than the private one.

Much empirical evidence, although not conclusive, indeed seems to suggest that the degree of positionality increases with the consumption level, such that relative consumption becomes more important compared to absolute consumption when the individuals get richer (see, e.g., Clark et al., 2008; Corazzini et al., 2012 and references therein).[9] This implies that, based on the present model, we may conclude that the social discount rate tends to exceed the private one under relative consumption effects.

Optimal consumption taxes

Although not the main purpose of the paper, it is worth mentioning that we can derive the set of consumption taxes that would internalize the time-dependent positional externalities. Let us assume a set of consumption taxes, such that q_t is the consumption tax at time t, where we normalize such that $q_0 = 0$, and where the tax revenues are distributed back in a lump-sum manner. We can then simply derive q_t as follows: if the individual is indifferent between undertaking and not undertaking a small project that will result in increased consumption at time t—which is paid for in terms of reduced consumption at time zero, and where consumption at time zero is untaxed and at time t is taxed by q_t—then the private

---

[8] Recall that $\rho^s(t)$ and $\rho^p(t)$ are discount rates defined over the discrete from time interval from time zero to time t, whereas v_{z_t} and v_{c_t} are local measures defined at time t. This means for example that it does not hold for all t that $\rho^s(t) = \rho^p(t)$ if and only if $v_{z_t} = \beta v_{c_t}$. For example, v_{z_t} may differ from βv_{c_t} in the time interval between 0.1t and 0.9t without violating the possibility that $\rho^s(t) = \rho^p(t)$.

[9] However, see Moav and Neeman (2012) for theoretical work showing that situations may arise where poor people are particularly positional, supported with some empirical evidence, which may contribute to a so-called poverty trap. Overall, although most evidence seems to suggest that the average degree of positionality tends to increase as consumption increases, there is no consensus on this issue.

discount rate per time unit is implicitly defined by

$$\frac{\partial w^p / \partial c_t}{\partial w^p / \partial c_0} = (1 + q_t) \exp(-\rho^p t),$$

implying that:

$$\rho^p = \delta - \frac{1}{t} \ln \left(\frac{v_{c_t}}{v_{c_0}} \frac{1}{1 + q_t} \right),$$

where the expression in parenthesis reflects the ratio of the marginal utility of own consumption, per monetary unit spent on consumption, between time t and time zero. If we set this private discount rate equal to the social one, given by Eq. (10), then:

$$q_t = \frac{\gamma_t - \gamma_0}{1 - \gamma_t}. \tag{11}$$

Thus, the tax is directly related to the difference in consumption positionality at time t, compared to the untaxed baseline at time zero. The intuition is straightforward, since γ_t is also a measure of social waste associated with consumption at time t. (Cf. with the corresponding static optimal tax results in Dupor and Liu, 2003; Aronsson and Johansson-Stenman, 2008.)

Comparisons with the conventional Ramsey discounting rule

In the previous section, we analyzed the conditions for when the social discount rate is lower or higher than the private one and concluded that the social discount rate exceeds the private one when the degree of positionality increases with the consumption level. We based this on a simple continuous time model where we discounted over an arbitrary discrete time period, from zero to t.

Yet, much of the discounting literature, including recent work on climate change, is based on the Ramsey discount rate, according to which the *instantaneous* discount rate consists of the pure rate of time preference plus the individual coefficient of relative risk aversion multiplied by the growth rate. It clearly has great policy relevance to compare the optimal social discount rate under relative consumption effects with the Ramsey discounting rule, which is the main purpose of this section.

Since the Ramsey discounting rule is normally derived over an infinitesimally short period of time,[10] here we solely consider instantaneous discount rates (hereafter discount rates). We will also compare the private and the social discount rates, as well as the private and the Ramsey discount rates, and hence provide conditions for ordering the sizes of the three discount rates.[11]

Three discount rates and three measures of relative risk aversion

We will undertake our analysis based on Eq. (1) (the same underlying instantaneous utility function as before), as well as Eqs. (2) and (3) (again the same time-consistent objective functions as before), and we will also keep the assumption of identical individuals with a population size normalized to one. Note first that, when t approaches zero, we can use first-order Taylor approximations to obtain $\partial w^p / \partial c_t = \partial w^p / \partial c_0 + t \partial (\partial w^p / \partial c_t) / \partial t$ and $\partial w^s / \partial c_t = \partial w^s / \partial c_0 + t \partial (\partial w^s / \partial c_t) / \partial t$. Substituting these into Eqs. (4) and (5), respectively, and applying l'Hôpital's rule, implies the following private and social instantaneous discount rates:

$$\rho_t^p = -\frac{\partial (\partial w^p / \partial c_t) / \partial t}{\partial w^p / \partial c_t} \tag{12}$$

$$\rho_t^s = -\frac{\partial (\partial w^s / \partial c_t) / \partial t}{\partial w^s / \partial c_t}. \tag{13}$$

Thus, ρ_t^p and ρ_t^s reflect the private and social discount rates in the time interval t to $t + dt$, whereas $\rho^p(t)$ and $\rho^s(t)$, analyzed in the previous section, reflect these discount rates in the discrete time interval from 0 to t.[12] In order to provide an economic interpretation to the comparison between these different instantaneous discount rates, we introduce some useful risk aversion definitions.

Definition 2. The individual coefficient of relative risk aversion is given by

$$\sigma_t \equiv -c_t v_{c_t c_t} / v_{c_t}. \tag{14}$$

---

[10] Alternatively, the formula can be derived over a discrete period of time if one is willing to assume constant relative risk aversion preferences, as well as a constant growth rate and pure rate of time preference. Here we do not want to limit ourselves to particular functional forms.

[11] For previous studies that have analyzed modifications of the Ramsey discounting rule, see, e.g., Hoel and Sterner (2007), Howarth (2009) and Gollier (2010).

[12] Of course, in the special case where ρ_t^p and ρ_t^s are both constant in the time interval from 0 to t, then $\rho^p(t) = \rho_t^p$ and $\rho^p(t) = \rho_t^s$.

The social coefficient of relative risk aversion is given by

$$\psi_t \equiv -c_t u_{c_t c_t} / u_{c_t}.\tag{15}$$

The coefficient of reference consumption relative risk aversion is given by

$$\theta_t \equiv z_t v_{z_t z_t} / v_{z_t}.\tag{16}$$

Thus, whereas σ_t reflects the conventional measure of relative risk aversion (or the individual's elasticity of marginal utility of increased consumption where others' consumption, z_t, is held fixed), ψ_t is a measure on the curvature of the instantaneous utility function, where relative consumption R_t is held fixed. It can therefore be thought of as a measure that reflects risk aversion had the individual made a risky choice *on behalf of the whole population*. Thus, it is the *social* coefficient of relative risk aversion. Finally, θ_t is a measure of the concavity, and hence risk aversion, with respect to reference consumption. Its definition is strictly analogous to the conventional measure of individual relative risk aversion, except that it refers to others' consumption. When $\theta_t > 0$, an individual prefers that others have consumption $\hat{z}_t$ with certainty over a situation where others' consumption is uncertain (with the expected value $\hat{z}_t$), whereas the individual is indifferent when $\theta_t = 0$.[13] By denoting the instantaneous growth rate at time t by $g_t \equiv \dot{c}_t / c_t$, where a dot denotes time derivative, we can now define the conventional instantaneous Ramsey discount rate as follows.

Definition 3. The discount rate according to the Ramsey discounting rule is given by

$$\rho_t^R = \delta + \sigma_t g_t.\tag{17}$$

Hence, the Ramsey discount rate simply consists of the pure rate of time preference plus the product of the individual coefficient of risk aversion times the growth rate, consistent with existing discounting literature.[14]

The relationship between the private and the Ramsey discount rate

From Eq. (2) follows that

$$-\frac{\partial (\partial w^p / \partial c_t) / \partial t}{\partial w^p / \partial c_t} = -\frac{v_{c_t c_t} \dot{c}_t + v_{c_t z_t} \dot{c}_t - \delta v_{c_t}}{v_{c_t}},$$

which substituted into Eq. (12), together with $g_t = \dot{c}_t / c_t$, implies the optimal private discount rate as follows[15]:

$$\rho_t^p = \delta - \frac{v_{c_t c_t}}{v_{c_t}} c_t g_t - \frac{v_{c_t z_t}}{v_{c_t}} c_t g_t,\tag{18}$$

By combining Eqs. (17) and (18) and using Definition 3, we then obtain:

$$\rho_t^p = \delta + \sigma_t g_t - \frac{v_{c_t z_t}}{v_{c_t}} c_t g_t = \rho_t^R - \frac{v_{c_t z_t}}{v_{c_t}} c_t g_t.\tag{19}$$

From Eq. (19) we immediately get the following result:

Proposition 3. *For $g_t > 0$, $\rho_t^p < \rho_t^R$.*

Thus, if the growth rate is positive and the preferences are characterized by the KUJ property, then the private discount rate falls short of the Ramsey discount rate. The intuition is that if my preferences are characterized by the KUJ property, then I will perceive future consumption relative to the present one as more valuable if others consume more in the future. Yet, this is the same as saying that my private discount rate will decrease as others' consumption grows.

The relationship between the private and the social discount rate

While we have already analyzed this relationship for the case of discounting over a discrete time period, let us for comparison purposes also derive corresponding expressions for the case of *instantaneous* discounting. From Eq. (3) follows that

$$-\frac{\partial (\partial w^s / \partial c_t) / \partial t}{\partial w^s / \partial c_t} = -\frac{v_{c_t c_t} \dot{c}_t + v_{c_t z_t} \dot{c}_t + v_{z_t c_t} \dot{c}_t + v_{z_t z_t} \dot{c}_t - \delta(v_{c_t} + v_{z_t})}{v_{c_t} + v_{z_t}},$$

---

[13] Note that reference consumption risk aversion ($v_{z_t z_t} < 0$) is neither necessary nor sufficient for the more frequently discussed concept of comparison concavity ($u_{R_t R_t} < 0$). See Clark and Oswald (1998) for some relationships between comparison-concavity and KUJ behavior.

[14] Yet, the measure of relative risk aversion will of course depend on the preferences, which here depend also on relative consumption. Still, the measure of relative risk aversion is defined due to changes for the individual alone, whether in terms of risky choices or consumption changes over time, which is how the measures are typically quantified empirically.

[15] It is easy to show that the same rule applies for discounting over a discrete time period (in continuous time), provided that ψ_t and g_t are constant in the interval considered. Recall also that since $z_t = c_t$ it follows that $\dot{z}_t = \dot{c}_t$.

which substituted into Eq. (13) implies the following optimal social discount rate:

$$\rho_t^s = \delta - \frac{v_{c_t c_t} + 2v_{c_t z_t} + v_{z_t z_t}}{v_{c_t} + v_{z_t}} c_t g_t = \delta + \sigma_t g_t - \frac{v_{c_t z_t}}{v_{c_t}} c_t g_t + \frac{d\gamma_t / d c_t}{1 - \gamma_t} c_t g_t, \tag{20}$$

where, in the second step, we have used

$$\frac{d\gamma_t}{dc_t} = -\left(\frac{v_{z_t z_t}}{v_{c_t}} + \frac{v_{z_t c_t}}{v_{c_t}} - \frac{v_{z_t}}{v_{c_t}^2} v_{c_t c_t} - \frac{v_{z_t}}{v_{c_t}^2} v_{c_t z_t} \right).$$

By comparing Eqs. (19) and (20), we then obtain[16]:

$$\rho_t^s = \rho_t^p + \frac{d\gamma_t / d c_t}{1 - \gamma_t} c_t g_t, \tag{21}$$

implying that we can again confirm Proposition 2.

In order to simplify this expression further, we can define *the degree of non-positionality* as

$$\varsigma_t \equiv 1 - \gamma_t = \frac{u_{c_t}}{u_{c_t} + u_{R_t} r_{c_t}}, \tag{22}$$

and the corresponding *consumption elasticity of non-positionality* as

$$\Gamma_t \equiv \frac{d\varsigma_t}{dc_t} \frac{c_t}{\varsigma_t}, \tag{23}$$

The degree of non-positionality is hence defined by the fraction of the overall utility increase from the last dollar consumed that is due to increased *absolute* consumption, whereas the consumption elasticity of non-positionality can be interpreted as (approximately) the percentage change in non-positionality that arises from a percentage consumption increase. We can then use Eq. (23) and rewrite Eqs. (20) and (21) as

$$\rho_t^s = \delta + \sigma_t g_t - \frac{v_{c_t z_t}}{v_{c_t}} c_t g_t - \Gamma_t g_t = \rho_t^p - \Gamma_t g_t. \tag{24}$$

Thus, the social discount rate is equal to the private one minus the consumption elasticity of non-positionality times the consumption growth rate. Note that $\Gamma_t g_t$ reflects the growth rate of the fraction of consumption that, from a social point of view, is not waste. If this growth rate is positive, it implies a reason for society to consume more in the future, implying a lower social discount rate. If it is negative, and more in line with the empirical evidence mentioned, then the opposite applies.

The relationship between the social and the Ramsey discount rate

Let us now turn to the most central part of this section, the comparison between the social and the Ramsey discount rate. So far, in Eq. (19) we expressed the relationship between the private and the Ramsey discount rate, in terms of a KUJ measure, and in Eq. (24) we expressed the relationship between the private and the social discount rate, in terms of how the degree of positionality depends on the consumption level. By combining Eqs. (19) and (24), we can clearly present the relationship between the social and the Ramsey discount rate, in terms of a KUJ measure and how the degree of positionality depends on the consumption level, as follows:

$$\rho_t^s = \rho_t^R - \frac{v_{c_t z_t}}{v_{c_t}} c_t g_t + \frac{d\gamma_t / d c_t}{1 - \gamma_t} c_t g_t = \rho_t^R - \frac{v_{c_t z_t}}{v_{c_t}} c_t g_t - \Gamma_t g_t. \tag{25}$$

Thus, according to this formulation, the social discount rate exceeds the Ramsey discount rate if the effects—through increased positionality (or decreased non-positionality) with consumption, and hence over time, given a positive growth rate—exceed the KUJ effect. Both assumptions are fairly intuitive and, if we accept them, we cannot say anything regarding the relative size of ρ_t^s and ρ_t^R without adding information regarding the relative strengths of these mechanisms. Yet, we will indirectly show conditions for when the KUJ effect dominates the increasing positionality effect.

In doing this, let us use the alternative formulation of Eq. (3), where w^s is expressed in terms of the u function. It then follows that

$$-\frac{\partial(\partial w / \partial c_t) / \partial t}{\partial w / \partial c_t} = -\frac{u_{c_t c_t} \dot{c}_t - \delta u_{c_t}}{u_{c_t}},$$

which substituted into Eq. (12) implies the following alternative formulation of the optimal social discount rate:

$$\rho_t^s = \delta + \psi_t g_t. \tag{26}$$

By comparing Eqs. (17) and (26), we immediately have:

---

[16] Alternatively, one could have derived this expression by letting t go to zero in Eq. (7) and applying l'Hôpital's rule on the second term.

Proposition 4. For $g_t > 0$, $\rho_t^s > (<)\rho_t^R$ if and only if $\psi_t > (<)\sigma_t$.

In words, given a positive growth rate, the welfare-maximizing social discount rate in the presence of relative consumption effects exceeds the conventional Ramsey discount rate if the social coefficient of relative risk aversion exceeds the individual coefficient of relative risk aversion, and vice versa. The next question is obvious. Under which conditions do ψ_t exceed σ_t, and vice versa? In order to interpret the conditions for this in economic terms, we introduce a measure of the complementarity between own and (reduction of) others´ consumption.

Definition 4. The elasticity of substitution between c_t and z_t is given by

$$\Phi_t \equiv \frac{-v_{c_t c_t}/v_{c_t}^2 - v_{z_t z_t}/v_{z_t}^2 + 2v_{c_t z_t}/(v_{c_t} v_{z_t})}{1/(v_{c_t} c_t) + 1/(v_{z_t} z_t)} = \left(-\frac{v_{c_t c_t}}{v_{c_t}^2} - \frac{v_{z_t z_t}}{v_{z_t}^2} + 2\frac{v_{c_t z_t}}{v_{c_t} v_{z_t}} \right) \frac{v_{c_t} v_{z_t}}{v_{c_t} + v_{z_t}} c_t. \tag{27}$$

This definition is standard (although it is not often used between one good and one bad in the utility function). It reflects the degree of quasi-concavity of v, and hence the degree of convexity of the indifference curves in c_t, z_t space. When $\Phi_t > 0$ everywhere, an individual would, roughly speaking, prefer averages to extremes also with respect to others' consumption. Thus, an individual who is indifferent between two different bundles of own and others' consumption A and B, given by $v(c_t^A, z_t^A)$ and $v(c_t^B, z_t^B)$ respectively, would thus always prefer any weighted average (linear combination) of A and B to each of the bundles A and B. $\Phi_t = 0$ everywhere would instead give linear indifference curves, such that the increase in consumption necessary to compensate an individual for other people's increase in consumption would be constant. We can now specify the following crucial relation between ψ_t and σ_t:

Lemma 1. The social coefficient of relative risk aversion is given by

$$\psi_t = \sigma_t - \theta_t - \Phi_t \tag{28}$$

Proof. Let us first express the social coefficient of relative risk aversion ψ_t in terms of the v function. By combining Eqs. (18) and (23), we have:

$$\psi_t = -\frac{v_{c_t c_t} + 2v_{c_t z_t} + v_{z_t z_t}}{v_{c_t} + v_{z_t}} c_t. \tag{29}$$

We can now combine Eqs. (14) and (29) and (after some straightforward algebraic manipulations) obtain:

$$\psi_t - \sigma_t = -c_t \frac{v_{z_t z_t}}{v_{z_t}} - c_t \frac{v_{c_t} v_{z_t}}{v_{c_t} + v_{z_t}} \left(-\frac{v_{c_t c_t}}{v_{c_t}^2} - \frac{v_{z_t z_t}}{v_{z_t}^2} + 2\frac{v_{c_t z_t}}{v_{c_t} v_{z_t}} \right). \tag{30}$$

Substituting, finally, Eqs. (16) and (27) into Eq. (30) gives Eq. (28).

Directly from Lemma 1 and Eq. (10), we can specify the social discount rate in terms of individual risk aversion as follows:

$$\rho_t^s = \delta + (\sigma_t - \theta_t - \Phi_t)g_t = \rho_t^R - (\theta_t + \Phi_t)g_t. \tag{31}$$

It is then straightforward to determine the conditions for when the social discount rate exceeds the one based on the conventional Ramsey discounting rule, and vice versa:

Proposition 5. For $g_t > 0$, $\rho_t^s < (>)\rho_t^R$ if $\theta_t + \Phi_t > (<)$ 0.

Thus, a sufficient, but not necessary, condition for $\rho_t^s < \rho_t^R$ is that v is strictly quasi-concave ($\Phi_t > 0$) and individuals are weakly reference-consumption risk averse ($v_{z_t z_t} \leq 0$, $\theta_t \geq 0$) or, alternatively, that v is weakly quasi-concave ($\Phi_t \geq 0$) and individuals are strictly reference-consumption risk averse ($v_{z_t z_t} < 0$, $\theta_t > 0$).

Overall, then, Proposition 5 says that the social discount rate with relative consumption concerns should be lower than the conventional Ramsey discount rate under what seem to be rather weak assumptions. What is the intuition behind that? Note first that the relative risk aversion measures can be interpreted as elasticities of marginal utility of consumption. As such, the elasticities roughly reflect the percentage decrease in marginal utility of consumption caused by a one percent consumption increase. Put in this way, a percentage increase in consumption will typically cause a larger percentage increase of a composite measure consisting of own consumption and (a fixed) reference consumption, if the reference consumption contributes to decrease the composite measure. Another way to express this is that an increase in c_t alone will positively affect both absolute and relative consumption, while a joint increase of both c_t and z_t will only increase absolute consumption. To see this more clearly, consider an instantaneous utility function as follows: $v(c_t, z_t) = f(c_t - az_t) = u(c_t, r(c_t, z_t)) = f((1-a)c_t + a(c_t - z_t))$, where $0 < a < 1$ clearly reflects the degree of positionality and where $f' > 0$ and $f'' < 0$. We may want to think of $c_t - az_t$ as *resulting consumption*, i.e., the consumption measure net of relative consumption concerns that affect instantaneous utility. From the first formulation we see that a one percent increase in c_t alone will cause a larger percentage increase in the resulting consumption ($c_t - az_t$) than what a one percent joint increase in both c_t and z_t would. This is perhaps even easier to see through the second formulation. Here, a one percent increase in c_t alone will increase both absolute consumption (c_t) and relative consumption ($c_t - z_t$), while a combined increase of c_t and z_t will only increase absolute consumption. Indeed, in the extreme case when a approaches one, a combined increase of c_t and z_t will not increase the resulting consumption at all, implying that utility as well as marginal utility will be held constant.

More formally, we have that $v_{c_t} = f'$, $v_{c_t c_t} = f''$, $u_{c_t} = (1-a)f'$, and $u_{c_t c_t} = (1-a)^2 f''$, implying that $\sigma_t = -c_t f''/f'$ and $\psi_t = -(1-a)c_t f''/f'$, such that $\psi_t - \sigma_t = a c_t f''/f' < 0$, and hence $\rho_t^s < \rho_t^R$. Thus, we have shown that the social discount rate is lower than the Ramsey discount rate when instantaneous utility is based on a difference comparison formulation (which is the most commonly used one in the literature) regardless of the function f. While this is a strong result in itself, Proposition 5 clarifies that this will not hold for all instantaneous utility functions.

Moreover, while the proposition also clarifies that the inequality holds under rather weak assumptions, the factors θ_t and Φ_t are less helpful in isolation when it comes to explaining the deviation between the individual and social coefficients of relative risk aversion, and hence between the Ramsey and the social discount rates. The reason is that these factors are neither independent of each other, nor of other measures reflecting curvatures and complementarities. Instead it turns out to be more illuminating to focus on the KUJ property.

For example, by combining equations (25) and (31) we obtain that

$$\theta_t + \Phi_t = \frac{v_{c_t z_t}}{v_{c_t}} c_t - \frac{d\gamma_t/dc_t}{1-\gamma_t} c_t,$$

and equations (25) in itself highlights that the KUJ effect ($v_{c_t z_t} > 0$) must be sufficiently strong to outweigh a possible effect of increasing degree of positionality, for the Ramsey discount rate to exceed the social one. It also shows that for the special case where the degree of positionality is constant and independent of the consumption level, σ_t exceeds ψ_t if and only if $v_{c_t z_t} > 0$. It is also straightforward to rewrite Eq. (30) as follows:

$$\psi_t - \sigma_t = \frac{\gamma_t}{1-\gamma_t} \left(\sigma_t + 2\frac{v_{c_t z_t}}{v_{c_t}} c_t + \theta \right). \tag{32}$$

Note that this is an implicit formulation, since σ_t appears on both sides. Nevertheless, this formulation also highlights the importance of the KUJ property, compared to the concavity with respect to own and others' consumption, respectively. Note also that the factor actually here contributes to the $\psi_t - \sigma_t$ difference in the opposite direction compared to in Eq. (31). This suggests that it is far from straightforward to interpret the contribution of risk aversion with respect to others' consumption (θ_t) in itself to the difference between ψ_t and σ_t. The importance of the KUJ property, on the other hand, is clear from both Eqs. (25) and (32).

Consider therefore the interpretation of Eq. (25), and let us start with the effect of the KUJ assumption. Intuitively, when my consumption increases my marginal utility of consumption will clearly decrease (whether others' consumption is held fixed or not). If also others' consumption increases (which is implied if relative consumption is held fixed), then, given KUJ, my marginal utility of consumption will decrease less than if others' consumption was fixed. Consequently, the KUJ property contributes to decrease ψ_t relative to σ_t, and as such the social discount rate relative to the Ramsey rate.

Consider finally the remaining element on the right-hand-side of Eq. (25), i.e., the effect through $d\gamma_t/dc_t$. The logic behind this term is similar to the comparison between the social and the private discount rates. If γ_t increases with consumption it will increase over time (given a positive growth rate), and, as such, it implies larger overall consumption waste in the future compared to now, implying an argument for a higher discount rate. The Ramsey discount rate does not reflect such positionality changes, implying that the effect through increased degree of positionality with consumption (and hence over time) contributes to increase the social discount rate relative to the Ramsey rate.

When the government does not respect individual preferences for social comparisons

So far we have consistently assumed that the government completely respects individual preferences, including concerns for relative consumption. However, Harsanyi (1982, p. 56) and others have argued that the government should not respect what might be seen as anti-social preferences, such as envy. While we are skeptical to this argument, it is nevertheless of interest to analyze how the results would change when the government does not respect individual concerns for relative consumption. What would that imply in our framework? A natural starting point would then be to assume that the government instead of maximizing a stream of discounted utilities maximizes the same stream while not taking into account any utility changes that are due to changed relative consumption. However, recall that in our welfarist framework we are holding relative consumption fixed in the social maximization problem, so there are no such utility changes to abstract from! Consequently, the social maximization problem will here remain the same as for the welfarist case, implying that the rule for the optimal social discount rate will also remain the same.

Moreover, the private discount rate will obviously not change either, since the individual will make choices over time independent of the objectives of the government. What about the Ramsey discount rate? This will also remain the same, since the individual coefficient of relative risk aversion (or elasticity of marginal utility of consumption) does not depend on the governmental objectives either. Together, perhaps surprisingly, this means that all results so far continue to hold under a non-welfarist government that does not respect individual preferences for social comparisons.[17] See also Eckerstorfer and

---

[17] Note that the government having a non-welfaristic objective function does not mean that it ignores effects of social comparisons, or that it does not observe that individuals care about social comparisons. If the government instead acts conditional on an (incorrect) assumption that individual instantaneous utility solely depends on absolute consumption, then there would consequently be no discrepancy between the social and the Ramsey discount rate.

Wendner (2013) and Aronsson and Johansson-Stenman (2014b) who in different optimal taxation frameworks with social comparisons have demonstrated that whether the government is welfarist or not may matter less than conjectured for optimal taxation rules.

Summary of main findings

By combining Eq. (20) and Proposition 5, we can summarize our main findings in this section regarding the ordering of our three different discount rates as follows[18]:

Corollary 1. *For $g_t > 0$, if $d\gamma_t/dc_t > 0$ and $\theta_t + \Phi_t > 0$, then $\rho_t^p < \rho_t^s < \rho_t^R$.*

Spelled out, this means that, for a positive consumption growth rate, the social discount rate is higher than the private rate, but lower than the Ramsey rate, if the degree of positionality increases with consumption and preferences reflect risk aversion with respect to reference consumption and are quasi-concave with respect to own and reference consumption. Briefly, the $\rho_t^p < \rho_t^s$ part is because if higher consumption in the future means higher positionality, then the corresponding waste will be higher too, and this waste is accounted for in the social but not the private discount rate. The intuition behind the $\rho_t^s < \rho_t^R$ part is less straightforward. First, $\theta_t + \Phi_t > 0$ implies that the KUJ-property is fulfilled (unless $d\gamma_t/dc_t < 0$). This property, in turn, implies that the marginal utility of consumption will generally decrease with consumption at a slower rate when others' consumption increases over time. This, in turn, is the case when others' consumption is held fixed, which is how the optimal social discount rate is calculated. A lower decrease in marginal utility of consumption, finally, means a lower social discount rate.

Note also that all three discount rates are identical when the degree of positionality is equal to zero, which is the standard case in the literature. Moreover, all discount rates are identical also for the case of a positive degree of positionality, provided that this degree is independent of the consumption level and that simultaneously $\theta_t = -\Phi_t$. Alternatively, all discount rates are identical if simultaneously $d\gamma_t/dc_t = 0$ and $v_{c_t z_t} = 0$, where the latter means that the marginal utility of own consumption is independent of the level of others' consumption (contrary to what is typically assumed). We have also shown that all results above continue to hold in the case of a non-welfarist government.

Numerical illustration and orders of magnitude

So far we have concluded that, under what may seem to be fairly plausible assumptions, the social discount rate tends to exceed the private discount rate (sections "Private and social discount rates" and "Comparisons with the conventional Ramsey discounting rule"), but to fall short of the conventional Ramsey discounting rule (section "Comparisons with the conventional Ramsey discounting rule"). The latter finding is particularly important from a policy perspective, for example in the discussion on climate change.

To get an indication of the magnitude of the effects found, we utilize the following simple, albeit quite flexible, functional form characterized by constant elasticity of substitution and constant relative risk aversion, similar to one used by Dupor and Liu (2003) as follows:

$$U_t = \frac{1}{1-\alpha}\left((1-a)c_t^{1-\omega} + a\left(c_t^{1-\omega} - z_t^{1-\omega}\right)\right)^{(1-\alpha)/(1-\omega)} = \frac{1}{1-\alpha}\left(c_t^{1-\omega} - az_t^{1-\omega}\right)^{(1-\alpha)/(1-\omega)} \tag{33}$$

This functional form has some convenient properties:

- The degree of positionality (see Definition 1) is constant and given by $\gamma_t = a$.
- The elasticity of substitution between own consumption and reference consumption (see Definition 4) is constant and equal to ω.
- The coefficient of reference-consumption relative risk aversion (see Definition 2) is constant and given by $\theta_t = (a\alpha - \omega)/(1-a)$, implying reference-consumption risk aversion for $\omega < a\alpha$.
- The individual and the social coefficients of relative risk aversion (see Definition 2) are also constant and given by $\sigma_t = (\alpha - \omega a)/(1-a)$ and $\psi_t = \alpha$, respectively, implying that:

$$\psi_t - \sigma_t = (\omega - \alpha)\frac{a}{1-a}, \tag{34}$$

  which is clearly weakly negative (such that $\rho_t^s \leq \rho_t^R$) as long as $\omega \leq \alpha$, which is required for the weak KUJ property assumed.
- The criterion that relative consumption is unaffected if own consumption and others' consumption are changed equally, i.e., the $r_{c_t} = -r_{z_t}$ assumption is fulfilled.

The constant elasticity of substitution functional form in Eq. (33) includes as special cases the two most commonly used comparison-consumption functional forms. When $\omega = 0$, we obtain the following simple *difference comparison form*, so that

---

[18] Note that if the conditions in Corollary 1 are fulfilled, then it also follows that $v_{c_t z_t} > 0$, i.e., the KUJ property is fulfilled.
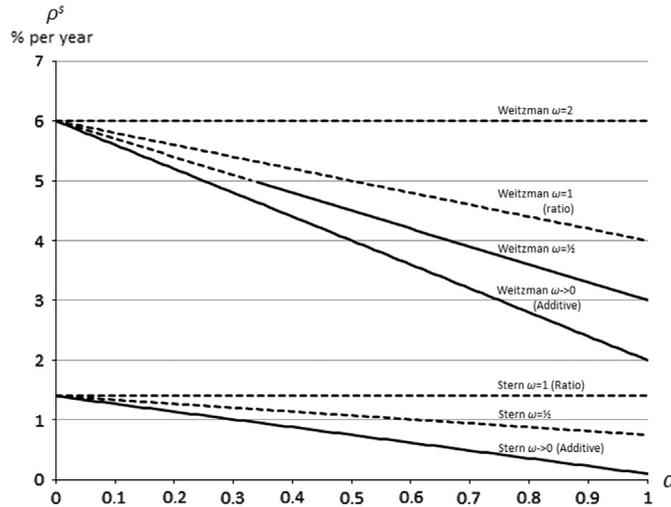
Fig. 1. The effect of the degree of postionality a on the optimal annual social discount rates based on the assumptions associated with Stern (2006) and Weitzman (2007b).

own consumption and (the negative of) others' consumption are perfect substitutes:

$$U_t = \frac{1}{1-\alpha}((1-a)c_t + a(c_t - z_t))^{(1-\alpha)} = \frac{1}{1-\alpha}(c_t - az_t)^{(1-\alpha)}. \tag{35}$$

For this functional form, we find that $\psi_t - \sigma_t = -\alpha a/(1-a) < 0$. For example, if $a = 0.5$ and $\alpha = 1$, then $\psi_t - \sigma_t = -1$, implying that the private coefficient of relative risk aversion $\sigma_t = 2$, whereas the corresponding social coefficient $\psi_t = 1$. Hence, it is clear that the effects of relative consumption may be substantial.

Similarly, we obtain the *ratio-comparison form* by letting ω approach unity and applying l'Hôpital's rule, implying that Eq. (33) converges to

$$U_t = \frac{1}{1-\alpha}\left(c_t\left(\frac{c_t}{z_t}\right)^{a/(1-a)}\right)^{(1-\alpha)} = \frac{1}{1-\alpha}c_t^{(1-\alpha)/(1-a)}z_t^{-(1-\alpha)a/(1-a)}, \tag{36}$$

such that $\psi_t - \sigma_t = (1-\alpha)a/(1-a)$, which is clearly negative if $\alpha > 1$ and positive if $\alpha < 1$. Note that reference-consumption risk aversion implies that $\alpha > 1/a$, and the KUJ assumption implies that $\alpha \geq 1$. Here, too, it is clear that the effects of relative consumption may be substantial as we illustrate further below. In order to assess orders of magnitude of the effects on the discount rates, we substitute Eq. (34) into Eq. (25) and obtain that

$$\rho_t^s = \delta + \left(\sigma + (\omega - \alpha)\frac{a}{1-a}\right)g = \delta + (\sigma - a(\sigma_t - \omega))g, \tag{37}$$

where in the letter stage we have replaced α by using the definition of σ. In Fig. 1, we plot the optimal social discount rate as a function of the degree of positionality for two commonly discussed sets of assumptions in the economic climate change literature, associated with Stern (2006) and Weitzman (2007b), respectively. Stern (2006) assumed that $\delta = 0.1$ (in percentage terms), $\sigma = 1$, and $g = 1.3$, implying an overall Ramsey discount rate of 1.4 percent annually,[19] whereas Weitzman (2007b) discusses the case where $\delta = \sigma = g = 2$,[20] leading to a Ramsey discount rate of 6 percent. Since both Stern and Weitzman conventionally assumed that relative consumption does not matter for utility, the corresponding social discount rates can be found to the left in the diagram where $a = 0$. As can be seen, how much relative consumption concerns affect the optimal social discount rate will then depend also on the elasticity of substitution, ω. We only plot the relationships for the case where the weak KUJ property is fulfilled, i.e., where $\omega \leq \alpha$, implying in the Stern case that $\omega \leq 1$ and in the Weitzman case that $\omega \leq 2$. Consistent with the results in Section "The relationship between the social and the Ramsey discount rate" (Eq. 25), the optimal social discount rate is then always equal to or below the Ramsey discount rate since the degree of positionality is constant.

We illustrate the parts of the relationships where the individual is reference-consumption risk averse[21] with solid lines, and the parts where the individual is reference-consumption risk-loving with dotted lines. Clearly, for non-negligible levels

---

[19] It may seem that Stern selected these values partly to compensate for a number of simplifications and omissions. Stern (2006) mentioned combinations of ethical, distributional issues, and deep uncertainty, as well as the effects of different growth rates in different sectors. Relative consumption issues were not mentioned, however.

[20] In personal communication, Weitzman recently told us he would personally favor a near-zero rate of pure time preference for climate change, and a coefficient of relative risk aversion of 2.5 or 3 rather than 2. Yet, he still agrees that the 2-2-2 rule, which was originally introduced more as a thought experiment, is still relevant since it is widely used by others in the climate change literature.

[21] It is easy to show that the individual is reference consumption risk averse if and only if $\omega < \sigma a(1+a)$.

of a, the optimal discount rate tends to be substantially lower than the Ramsey rate if we assume reference-consumption risk aversion.

Similar patterns hold for most other reasonable parameter combinations. For example, in the macro-economic literature one often assumes, drawing on Hall (1988), that $\sigma = 5$ and $g = 1.8$. Ignoring the pure rate of time preferences (where any value can simply be added to the social discount rate), the social discount rate based on $\omega = 2$ is then reduced from 9 percent at $a = 0$ to 6.3 percent per annum at $a = 0.5$. Correspondingly, for $\omega = 0.5$ the social discount rate is reduced to 4.95 percent per annum at $a = 0.5$.

Unfortunately, the empirical literature does not provide any precise estimates of the relevant parameters. The most central one, γ (or a), is obviously difficult to measure, and it is therefore not surprising that the available estimates vary considerably. However, most estimates are substantially above zero. According to the survey-experimental evidence of Solnick and Hemenway (1998, 2005), Johansson-Stenman et al. (2002), Alpízar et al. (2005), and Carlsson et al. (2007), the average degree appears to be in the order of magnitude of 0.5. Wendner and Goulder (2008) argued, based on existing empirical evidence, for a value between 0.2 and 0.4, whereas evidence from happiness studies, such as Luttmer (2005), suggests a much larger value close to unity. To our knowledge, there are no quantitative estimates of either θ or Φ—i.e., of either reference-consumption risk aversion or the degree of quasi-concavity between own consumption and reference consumption—beyond what is implied by KUJ behavior.

In contrast, there are many studies trying to estimate σ, which is relatively less important for the results here.[22] Estimates of δ are highly controversial, in particular for ethical reasons, when dealing with intergenerational issues (Stern, 2006). Yet, as shown above, δ does not affect the difference between the social and the Ramsey discount rates. Future growth rates are of course also difficult to predict.

Overall, the discrepancy between the social and the Ramsey discount rates due to relative consumption effects is clearly difficult to quantify, but may well be substantial and could even exceed 1–2 percentage points in the estimates that are most frequently used in the climate literature.

Conclusion and discussion

There are several reasons based on the existing literature why one may argue that social discount rates should in practice be lower than individual ones: individuals are more risk averse than society in the presence of uncertainty, and societal time horizons are longer than individual ones (cf. Arrow and Lind, 1970). In the present paper, we show that relative consumption effects do not provide another reason. On the contrary, the social discount rate under positional concern tends to exceed the private one, provided that the degree of positionality increases as we get richer and consumption increases (for which there is some empirical evidence).

Yet, from a climate policy perspective, it is presumably more important whether the optimal social discount rate should be modified compared to the one corresponding to the conventionally used discounting rule, i.e. the so-called Ramsey discounting rule. We show that for a positive growth rate, the social discount rate is lower than the Ramsey discount rate if preferences are quasi-concave in own and reference consumption (consisting of others' average consumption) and concave in reference consumption.

We also demonstrate numerically that the discrepancies may be substantial, although the underlying parameter estimates are highly uncertain. Since the impacts of the discount rates on the economics of long-term phenomena, such as global warming, are so large—even for modest adjustments of the discount rate—it is fair to conclude that taking relative consumption effects into account may have a profound effect on the economics of phenomena like global warming. The simulations moreover reveal that relative consumption effects may quantitatively be at least as important as adjusting for uncertainty (although the latter of course depends on the magnitudes of the uncertainty and the probability for catastrophic effects); cf. Arrow et al. (2013) for values of comparable magnitude and Gollier (2012) for a more general discussion.

Finally, it is worth emphasizing that the identified effects of relative consumption concerns do not at all depend on assumptions of inequality, or aversion to inequality, since the model is throughout the paper based on identical individuals who in equilibrium consume the same amount in each time period. We have also explained that all results continue to hold in the case of a non-welfarist government that does not respect individual preferences for social comparisons. Nevertheless, it would of course be interesting in future research to analyze how the results here are modified in alternative settings and contexts.

Acknowledgments

---

[22] For example, Friend and Blume (1975) concluded that σ generally exceeds unity and is probably greater than 2. Blundell et al. (1994) and Attanasio and Browning (1995) found, in most of their estimates, an order of magnitude of 1 or slightly above, whereas Vissing-Jørgensen (2002) found that σ differs between stockholders (approximately 2.5–3) and bond holders (approximately 1–1.2). Halek and Eisenhauer (2001) estimated values for a large sample of individuals and found a very skewed distribution with a mean over 3, but a median of 0.9.

References

Abel, A.B., 2005. Optimal taxation when consumers have endogenous benchmark levels of consumption. Rev. Econ. Stud. 72, 1–19.

Akerlof, G.A., 1997. Social distance and social decisions. Econometrica 65, 1005–1027.

Alpízar, F., Carlsson, F., Johansson-Stenman, O., 2005. How much do we care about absolute versus relative income and consumption? J. Econ. Behav. Organ. 56, 405–421.

Aronsson, T., Johansson-Stenman, O., 2008. When the Joneses' consumption hurts: optimal public good provision and nonlinear income taxation. J. Public Econ. 92, 986–997.

Aronsson, T., Johansson-Stenman, O., 2010. Positional concerns in an OLG model: optimal labor and capital income taxation. Int. Econ. Rev. 51, 1071–1095.

Aronsson, T., Johansson-Stenman, O., 2014a. State-variable public goods and social comparisons. J. Environ. Econ. Manage. 68, 390–410.

Aronsson, T., Johansson-Stenman, O., 2014b. Paternalism against Veblen: Optimal Taxation and Non-Respected Preferences for Social Comparisons. Working paper. Department of Economics, University of Gothenburg.

Arrow, K.J., Dasgupta, P.S., 2009. Conspicuous consumption, inconspicuous leisure. Econ. J. 119, F497–F516.

Arrow, K.J., Cline, W.R., Mäler, K.-G., Munasinghe, M., Squitieri, R., Stiglitz, J., 1996. Intertemporal equity, discounting, and economic efficiency. In: Bruce, J. P., Lee, H., Haites, E.F. (Eds.), Climate Change 1995: Economic and Social Dimensions—Contribution of Working Group III to the Second Assessment Report of the Intergovernmental Panel on Climate Change, Cambridge University Press, Cambridge, pp. 125–144.

Arrow, K., Cropper, M.L., Gollier, C., Groom, B., Heal, G.M., Newell, R.G., Nordhaus, W.D., Pindyck, R.S., Pizer, W.A., Portney, P., Sterner, T., Tol, R., Weitzman, M.L., 2013. Determining benefits and costs for future generations. Science 341, 349–350.

Arrow, K.J., Lind, R.C., 1970. Uncertainty and the evaluation of public investment decisions. Am. Econ. Rev. 6, 364–378.

Attanasio, O.P., Browning, M., 1995. Consumption over the life cycle and over the business cycle. Am. Econ. Rev. 85, 1118–1137.

Blanchflower, D.G., Oswald, A.J., 2004. Well-being over time in Britain and the USA. J. Public Econ. 88, 1359–1386.

Blundell, R., Browning, M., Meghir, C., 1994. Consumer demand and the life-cycle allocation of household expenditures. Rev. Econ. Stud. 61, 57–80.

Boskin, M.J., Sheshinski, E., 1978. Optimal redistributive taxation when individual welfare depends upon relative income. Q. J. Econ. 92, 589–601.

Bowles, S., Park, Y.-J., 2005. Inequality, emulation, and work hours: was Thorsten Veblen right? Econ. J. 15, F397–F413.

Brekke, K.A., Howarth, R., 2002. Status growth and the environment: goods as symbols in applied welfare economics. Ecol. Econ. 49, 414–415.

Brekke, K.A., Johansson-Stenman, O., 2008. The behavioral economics of climate change. Oxf. Rev. Econ. Policy 24, 280–297.

Cameron, H., Koundouri, P., Panopoulou, E., Pantelidis, T., 2009. Social discounting under uncertainty: a cross-country comparison. J. Environ. Econ. Manage. 57, 140–150.

Carlsson, F., Johansson-Stenman, O., Martinsson, P., 2007. Do you enjoy having more than others? Survey evidence of positional goods. Economica 74, 586–598.

Carroll, C.D., Overland, J., Weil, D.N., 1997. Comparison utility in a growth model. J. Econ. Growth 2, 339–367.

Clark, A.E., Frijters, P., Shields, M.A., 2008. Relative income, happiness, and utility: an explanation for the easterlin paradox and other puzzles. J. Econ. Lit. 46, 95–144.

Clark, A.E., Oswald, A.J., 1998. Comparison-concave utility and following behavior in social and economic settings. J. Public Econ. 70, 133–155.

Corazzini, L., Esposito, L., Majorano, F., 2012. Reign in hell or serve in heaven? A cross-country journey into the relative vs. absolute perceptions of wellbeing. J. Econ. Behav. Organ. 3, 715–730.

Dasgupta, P., 2007. Commentary: the stern review's economics of climate change. Natl. Inst. Econ. Rev. 199, 47.

Dasgupta, P., 2008. Discounting climate change. J. Risk Uncertain. 37, 141–169.

Dietz, S., Stern, N., 2007. Why economic analysis supports strong action on climate change: a response to the stern review's critics. Rev. Environ. Econ. Policy 2, 94–113.

Duesenberry, J.S., 1949. Income, Savings, and the Theory of Consumer Behavior. Harvard University Press, Cambridge, MA, USA.

Dupor, B., Liu, W.F., 2003. Jealousy and overconsumption. Am. Econ. Rev. 93, 423–428.

Eckerstorfer, P., Wendner, R., 2013. Asymmetric and non-atmospheric consumption externalities, and efficient consumption taxation. J. Public Econ. 106 (C), 42–56.

Ferrer-i-Carbonell, A., 2005. Income and well-being: an empirical analysis of the comparison income effect. J. Public Econ. 89, 997–1019.

Fliessbach, K., Weber, B., Trautner, P., Dohmen, T., Sunde, U., Elger, C.E., Falk, A., 2007. Social comparison affects reward-related brain activity in the human ventral striatum. Science 318, 1305–1308.

Frederick, S., Loewenstein, G., O'Donoghue, T., 2002. Time discounting and time preference: a critical review. J. Econ. Lit. 40, 351–401.

Friend, I., Blume, M.E., 1975. The demand for risky assets. Am. Econ. Rev. 65, 900–922.

Gali, J., 1994. Keeping up with the joneses: consumption externalities, portfolio choice, and asset prices. J. Money Credit Bank. 26, 1–8.

Gollier, C., 2010. Ecological discounting. J. Econ. Theory 145, 812–829.

Gollier, C., 2012. Pricing the future: the economics of discounting in an uncertain world. Princeton University Press, Princeton, New Jersey.

Gollier, C., Weitzman, M.L., 2010. How should the distant future be discounted when discount rates are uncertain? Econ. Lett. 107, 350–353.

Halek, M., Eisenhauer, J.G., 2001. Demography of risk aversion. J. Risk Insur. 68, 1–24.

Hall, R.E., 1988. Intertemporal substitution in consumption. J. Polit. Econ. 96, 339–357.

Harsanyi, J., 1982. Morality and the theory of rational behavior. In: Sen, A., Williams, B. (Eds.), Utilitarianism and Beyond, Cambridge University Press, Cambridge.

Heal, G., 2008. Climate change economics: a meta-review and some suggestions. Columbia University, School of International and Public Affairs, Faculty Research on Energy and Development, New York ⟨http://www2.gsb.columbia.edu/faculty/gheal/Climate-Change-Review.pdf⟩.

Hoel, M., Sterner, T., 2007. Discounting and relative prices. Clim. Change 84, 265–280.

Howarth, R.B., 2009. Discounting, uncertainty, and revealed time preference. Land Econ. 85, 24–40.

Johansson-Stenman, O., Carlsson, F., Daruvala, D., 2002. Measuring future grandparents' preferences for equality and relative standing. Econ. J. 112, 362–383.

Karp, L., Tsur, Y., 2011. Time perspective and climate change policy. J. Environ. Econ. Manage. 62, 1–14.

Layard, R., 1980. Human satisfaction and public policy. Econ. J. 90, 737–750.

Ljungqvist, L., Uhlig, H., 2000. Tax policy and aggregate demand management under catching up with the Joneses. Am. Econ. Rev. 90, 356–366.

Luttmer, E.F.P., 2005. Neighbors as negatives: relative earnings and well-being. Q. J. Econ. 120, 963–1002.

Moav, O., Neeman, Z., 2012. Saving rates and poverty: the role of conspicuous consumption and human capital. Econ. J. 563, 933–956.

Ng, Y.-K., 1987. Relative income effects and the appropriate level of public expenditure. Oxf. Econ. Pap. 39, 293–300.

Nordhaus, W.D., 2007a. A review of the stern review on the economics of climate change. J. Econ. Lit. 45, 686–702.

Nordhaus, W.D., 2007b. Critical assumptions in the stern review on climate change. Science 317, 201–202.

Oswald, A., 1983. Altruism, jealousy, and the theory of optimal non-linear taxation. J. Public Econ. 20, 77–87.

Ramsey, F., 1928. A mathematical theory of saving. Econ. J. 38, 543–549.

Rayo, L., Becker, G.S., 2007. Evolutionary efficiency and happiness. J. Polit. Econ. 115, 302–337.

Samuelson, L., 2004. Information-based relative consumption effects. Econometrica 72, 93–118.

Solnick, S., Hemenway, D., 2005. Are positional concerns stronger in some domains than in others? Am. Econ. Rev. 95, 147–151.

Solnick, S., Hemenway, D., 1998. Is more always better? A survey on positional concerns. J. Econ. Behav. Organ. 37, 373–383.

Stern, N., 2006. The Economics of Climate Change: The Stern Review. Cambridge University Press, Cambridge.

Sterner, T., Persson, U.M., 2008. An Even Sterner review: introducing relative prices into the discounting debate. Rev. Environ. Econ. Policy 2, 61–76.

Stevenson, B., Wolfers, J., 2008. Economic growth and subjective well-being: reassessing the Easterlin paradox. Brookings Papers on Economic Activity 2008, 88–102.

Vissing-Jørgensen, A., 2002. Limited asset market participation and the elasticity of intertemporal substitution. J. Polit. Econ. 4, 825–853.

Weitzman, M., 2007a. Subjective expectations of asset price puzzles. Am. Econ. Rev. 97, 1102–1130.

Weitzman, M., 2007b. A review of the stern review on the economics of climate change. J. Econ. Lit. 45, 703–724.

Weitzman, M., 2010. Risk-adjusted gamma discounting. J. Environ. Econ. Manage. 60, 1–13.

Wendner, R., 2010a. Growth and keeping up with the Joneses. Macroecon. Dyn. 14 (S2), 176–199.

Wendner, R., 2010b. Conspicuous consumption and generation replacement in a model of perpetual youth. J. Public Econ. 94, 1093–1107.

Wendner, R., 2011. Will the consumption externalities' effects in the Ramsey model please stand up? Econ. Lett. 111, 210–212.

Wendner, R., Goulder, L.H., 2008. Status effects, public goods provision, and the excess burden. J. Public Econ. 92, 1968–1985.