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State-variable public goods and social comparisons

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ABSTRACT

The optimal provision of a state-variable public good, where the global climate is the prime example, is analyzed in a model where people care about their relative consumption. We consider both keeping-up-with-the-Joneses preferences (where people compare their own current consumption with others' current consumption) and catching-up-with-the-Joneses preferences (where people compare their own current consumption with others' past consumption) in an economy with two productivity types, overlapping generations, and optimal nonlinear income taxation. The extent to which the conventional rules for provision of state-variable public goods (a dynamic analog of the Samuelson rules) ought to be modified is shown to clearly depend on the strength of the relative concerns of both kinds, but also on the preference elicitation format.

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Introduction

How much should we spend to combat climate change? Naturally, the answer depends on many factors such as the links between environmental damage and abatement investments, discounting issues, and on the nature of our preferences. It also depends on how the funds for the investment are raised and on related second-best problems of the economy, including how such investments interact with the use of other policy instruments. The present paper concerns the optimal provision rule for a state-variable public good, such that the public good can be seen as a stock that changes over time, in a dynamic economy where people have positional preferences for private consumption. The latter means that people derive utility from their own private consumption relative to that of others – a notion with solid support in both questionnaire-based experiments and happiness research.¹

The problem of characterizing the optimal provision of public goods under relative consumption concerns is not new in itself; it has been analyzed in a static setting with lump-sum taxes by Ng (1987), Brekke and Howarth (2002), and Aronsson and Johansson-Stenman (2014a), and in a static setting with second best taxation by Wendner and Goulder (2008), Aronsson and Johansson-Stenman (2008, 2013a), and Wendner (2014). Howarth (1996, 2006) analyzed the related problem of dealing with environmental externalities under relative consumption concerns.

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¹ See, e.g., Easterlin (1995, 2001), Solnick and Hemenway (1998, 2005), Johansson-Stenman et al. (2002), Blanchflower and Oswald (2004), Stutzer (2004), Alpizar et al. (2005), Ferrer-i-Carbonell (2005), Carlsson et al. (2007), Clark et al. (2008), Clark and Senik (2010), and Corazzini et al. (2012). See also evidence from brain science (Fliessbach et al., 2007; Dohmen et al., 2011).

As a consequence of using static models, all these studies have focused on cases where the public good (or bad) is a flow variable, and where the concept of relative consumption lacks a time-dimension. To our knowledge, the optimal provision of public goods under relative consumption concerns in a second-best economy has never been analyzed in a dynamic context, although Brekke and Howarth (2002), Wendner (2003, 2005), and Howarth (2006) have examined the implications of environmental externalities in dynamic models where relative consumption matters.² There is also a sizable literature on economic growth in models with relative consumption concerns (e.g., Konrad, 1992; Rauscher, 1997; Corneo and Jeanne, 1999; Brekke and Howarth, 2002; Abel, 2005; Wendner 2010a, 2010b). Since this dynamic public good provision problem obviously is more complex than its static counterpart, one may wonder whether the value added in terms of insights from such a study is worth the cost of additional complexity. We argue that it is for at least two reasons.

First, most public goods share important state-variable characteristics, in the sense that their quality depends on previous actions, where the global climate stands out as a prime example. Indeed, the question concerning how much we should invest to combat climate change is one of the most important and discussed of our times. The current quality, and characteristics more generally, of the atmosphere are clearly not only affected by the actions taken today (such as current public abatement activities); they are also strongly affected by actions taken in previous periods (cf. Stern, 2007). Correspondingly, actions taken today will affect the atmosphere for a long time. Similar arguments apply to many other public goods as well, including infrastructural investments such as roads, schools, and hospitals. It is therefore of high policy relevance to identify optimal provision rules for state-variable public goods, and to understand how these rules are modified due to relative consumption concerns, not least due to the important policy implications of such concerns found in other literature on taxation and public expenditure. Given the dynamic nature of the policy problem analyzed below, the paper will also, implicitly, touch on the discounting problem; yet, this is not the main task, and we will not discuss any intergenerational equity issues.³

Second, the dynamic framework allows us to examine the implications of a broader set of measures of relative consumption by considering comparisons based on *keeping-up-with-the-Joneses* preferences, where individuals compare their current consumption with other people's current consumption, simultaneously with intertemporal consumption comparisons. Such intertemporal comparisons may include comparisons with one's own past consumption as well as comparisons with other people's past consumption, where the latter will be referred to as *catching-up-with-the-Joneses*.⁴ The extension to intertemporal social comparisons is important for several reasons: (i) There is empirical evidence suggesting that people make comparisons with their own past consumption and with the past consumption of others.⁵ (ii) Intertemporal social comparisons have been found to have important implications for optimal tax policy (Ljungqvist and Uhlig, 2000; Aronsson and Johansson-Stenman, 2014b),⁶ and are also consistent with the equity premium puzzle discussed in the literature on dynamic macroeconomics (e.g., Abel, 1990; Campbell and Cochrane, 1999). (iii) Finally, such comparisons are also in line with recent research based on evolutionary models, such as Rayo and Becker (2007), in which there are evolutionary reasons for why people should compare their own current consumption with three distinct reference points: others' current consumption, their own past consumption, and others' past consumption. Our study relates to Rayo and Becker in the sense that we consider all three comparisons simultaneously in terms of their implications for public good provision. In addition to the value of identifying how the optimal public good provision rule should be modified due to these extensions, we argue that it is equally important to identify the extent to which the basic insights from static models of public good provision under relative consumption concerns carry over to the dynamic case with state-variable public goods. While some of the results derived below are similar to those found in static models, others are distinctly different.

Policy rules for public goods depend on the set of tax instruments that the government has at its disposal. The model in the present paper builds on the models developed in Aronsson and Johansson-Stenman (2010, 2014b), which address optimal income taxation under asymmetric information in an Overlapping Generations (OLG) framework with two ability-types but do not consider public goods, the concern of the present paper.⁷ Such a framework gives a reasonably realistic description of the information constraints inherent in redistribution policy; it also allows us to capture redistributive and corrective aspects of public good provision, as well as interaction effects between them, in a relatively simple way.

² Chapter 7 in the very rich book by Brekke and Howarth (2002) presents a model with stock externalities that has similarities with the model in the present paper. Public goods are also briefly discussed in a dynamic context, but the model is very different otherwise and is based on a first-best setup.

³ The choice of discount rate is perhaps the most discussed issue in the economics of global warming; see, e.g., Nordhaus (2007), Stern (2007), Weitzman (2009, 2010), and Milner (2013). See Johansson-Stenman and Sterner (2012) for an analysis of the optimal social discounting rule when relative consumption matters.

⁴ Other literature sometimes uses the concepts of "keeping-up-with-the-Joneses" and "catching-up-with-the-Joneses" in a more specific way based on assumptions about how the individual reacts to changes in others' current and previous consumption; see, e.g., Alvarez-Cuadrado et al. (2004) and Wendner (2010a, 2010b).

⁵ See, e.g., Loewenstein and Sicherman (1991) and Frank and Hutchens (1993) for evidence suggesting that people make comparisons with their own past consumption. Senik (2009) presents further empirical evidence pointing at the importance of historical benchmarks. She finds that individual well-being increases if the standard of living of the respondent's household increases compared with an internal benchmark given by the household's standard of living 15 year ago, and if the individual has done better in life than his/her parents, *ceteris paribus*.

⁶ Other literature on optimal taxation under relative consumption concerns typically focuses on atemporal (keeping-up-with-the-Joneses) comparisons; see, e.g., Boskin and Sheshinski (1978), Oswald (1983), Ireland (2001), and Dupor and Liu (2003).

⁷ The seminal paper on public good provision under optimal nonlinear income taxation is Hylland and Zeckhauser (1979), whereas Boadway and Keen (1993) was the first study dealing with this problem based on the self-selection approach to optimal taxation developed by Stern (1982) and Stiglitz (1982). To our knowledge, Pirttilä and Tuomala (2001) were the first to address the optimal provision of a state-variable public good under nonlinear taxation.

The section “[The OLG economy and individual preferences](#)” presents the OLG framework, preference structure and individual optimization problems, while the section “[Firm behavior](#)” considers the optimization problems of firms. In the section “[The government’s optimization problem](#)”, we describe the corresponding optimization problem facing the government. The section “[A general rule for public good provision when relative consumption matters](#)” presents rather general expressions for the optimal provision rule, which are valid for all kinds of social comparisons. Yet, while these results provide general insights on the incentives for public provision under relative consumption concerns, they are not directly interpretable in terms of the strength of such concerns. Therefore, the provision rules derived in the sections “[Optimal provision rules with keeping-up-with-the-Joneses preferences](#)” and “[Optimal provision of the public good under both keeping-up-with-the-Joneses and catching-up-with-the-Joneses preferences](#)” are expressed directly in terms of the degrees of positionality.

The section “[Optimal provision rules with keeping-up-with-the-Joneses preferences](#)” concerns the case where the individual only compares his/her own current consumption with other people’s current consumption (keeping-up-with-the-Joneses preferences); as such, it builds on the model by [Aronsson and Johansson-Stenman \(2010\)](#) and extends it to encompass public goods. The results here are shown to depend crucially on the preference elicitation format. If people’s marginal willingness to pay for the public good is measured independently, i.e., without considering that other people also have to pay for increased public provision, then relative consumption concerns typically (for reasonable parameter values) work in the direction of increasing the optimal provision of the public good. However, this is not the case when a referendum format is used, so that people are asked for their marginal willingness to pay conditional on that all people will have to pay for increased public provision. Conditions are also presented for when a dynamic analog of the conventional [Samuelson \(1954\)](#) rule applies.

The section “[Optimal provision of the public good under both keeping-up-with-the-Joneses and catching-up-with-the-Joneses preferences](#)” considers the more general case with both keeping-up and catching-up-with-the-Joneses preferences simultaneously, i.e., where people derive utility from their own consumption relative to the current *and* past consumption of others, as well as relative to their own past consumption; thus, the model extends the one in [Aronsson and Johansson-Stenman \(2014b\)](#) to encompass public goods. Under some further simplifying assumptions (e.g., about the population size and how the concerns for relative consumption change over time), it is shown that the policy rule for public provision can be written as a straightforward extension of the corresponding policy rule derived solely on the basis of keeping-up-with-the-Joneses preferences in the section “[Optimal provision rules with keeping-up-with-the-Joneses preferences](#)”. However, contrary to the findings in the section “[Optimal provision rules with keeping-up-with-the-Joneses preferences](#)”, we also show that if individuals compare their own consumption with other people’s past consumption, a referendum format for measuring marginal benefits conditional on that others also have to pay for the public good no longer leads to a straightforward dynamic analog to the Samuelson condition. The section “[Conclusion](#)” provides some concluding remarks. Proofs of all propositions (along with some other calculations) are provided in the [Appendix](#).

The OLG economy and individual preferences

We consider an economy where individuals live for two periods. An individual of generation t is young in period t and old in period $t+1$. We assume that each individual works during the first period of life and does not work during the second. Individuals differ in ability (productivity) and we simplify by considering a framework with two ability types, where the low-ability type (type 1) is less productive than the high-ability type (type 2). Each individual of ability-type i and generation t cares about his/her consumption when young and when old, c_t^i and x_{t+1}^i ; his/her leisure when young, z_t^i ; and the amount of the public good available (which can be interpreted as the quality of the climate) when young and when old, G_t and G_{t+1} . The individual also derives utility through his/her relative consumption by comparison with (a) other people’s current consumption, (b) his/her own past consumption, and (c) other people’s past consumption. This is soon to be explained more thoroughly.

The lifetime utility function faced by ability-type i of generation t is written as follows:

$$\begin{aligned} U_t^i &= V_t^i(c_t^i, z_t^i, x_{t+1}^i, c_t^i - \bar{c}_t, x_{t+1}^i - \bar{c}_{t+1}, x_{t+1}^i - c_t^i, c_t^i - \bar{c}_{t-1}, x_{t+1}^i - \bar{c}_t, G_t, G_{t+1}) \\ &= v_t^i(c_t^i, z_t^i, x_{t+1}^i, c_t^i - \bar{c}_t, x_{t+1}^i - \bar{c}_{t+1}, c_t^i - \bar{c}_{t-1}, x_{t+1}^i - \bar{c}_t, G_t, G_{t+1}) \\ &= u_t^i(c_t^i, z_t^i, x_{t+1}^i, \bar{c}_{t-1}, \bar{c}_t, \bar{c}_{t+1}, G_t, G_{t+1}), \end{aligned} \quad (1)$$

where $\bar{c}_t = \sum_i [n_t^i c_t^i + n_{t-1}^i x_t^i] / \sum_i [n_t^i + n_{t-1}^i] = \sum_i [n_t^i c_t^i + n_{t-1}^i x_t^i] / N_t$ denotes the average consumption in the economy as a whole in period t ; n_t^i measures the number of young individuals of ability-type i in period t (implying, of course, that n_{t-1}^i represents the number of old individuals of ability-type i in period t) and $N_t \equiv \sum_i [n_{t-1}^i + n_t^i]$ denotes the total population in period t . The five consumption differences in Eq. (1) – as represented by the fourth to eighth argument in the function $V^i(\cdot)$ – are measures of relative consumption, and imply that the individual compares his/her current consumption with (a) the current average consumption when young and when old, i.e., $c_t^i - \bar{c}_t$ and $x_{t+1}^i - \bar{c}_{t+1}$; (b) his/her own consumption one period earlier when old, i.e., $x_{t+1}^i - c_t^i$; and (c) the average consumption one period earlier when young and when old, i.e., $c_t^i - \bar{c}_{t-1}$ and $x_{t+1}^i - \bar{c}_t$. Two things are worth noting. First, the relative consumption is defined as the *difference* between the individual’s own consumption and the appropriate reference measure; this approach is technically convenient and has been used in many previous studies (e.g., [Akerlof, 1997](#); [Corneo and Jeanne, 1997](#); [Ljungqvist and Uhlig, 2000](#); [Bowles and Park,](#)

2005; Carlsson et al.; 2007).⁸ Second, the measures of reference consumption implicit in comparisons (a) and (c) are indicators of the average consumption in the economy as a whole, which is the common way to define reference consumption in earlier studies.⁹

To explain the second utility formulation in Eq. (1), i.e., $v_t^i(\cdot)$, note that c_t^i and x_{t+1}^i are decision variables of the individual; therefore, we can without loss of generality use the simpler function $v_t^i(\cdot)$ on the second line, where the effect of $x_{t+1}^i - c_t^i$ on utility is embedded in the effects of c_t^i and x_{t+1}^i . As such, habit formation does not produce a corrective motive for provision of public goods. The function $u_t^i(\cdot)$ is a convenient reduced form to be used in some of the calculations presented below; however, $u_t^i(\cdot)$ also represents the most general utility formulation in the sense of not specifying how the relative consumption comparisons are made (other than that others' consumption gives rise to negative externalities).

The policy rules for public provision examined below reflect the extent to which relative consumption concerns are important for individual well-being. As such, it is useful to measure the degree to which such concerns matter for each individual, which we will do by employing the second utility formulation, $v_t^i(\cdot)$, in Eq. (1). By using the variables

$$\Delta_t^{i,c} \equiv c_t^i - \bar{c}_t, \Delta_{t+1}^{i,x} \equiv x_{t+1}^i - \bar{c}_{t+1}, \delta_t^{i,c} \equiv c_t^i - \bar{c}_{t-1}, \text{ and } \delta_{t+1}^{i,x} \equiv x_{t+1}^i - \bar{c}_t$$

as short notations for the four differences in the function $v_t^i(\cdot)$, we follow Aronsson and Johansson-Stenman (2014b) and define the *degree of current consumption positionality* for ability-type i of generation t when young and old, respectively, as

$$\alpha_t^{i,c} \equiv \frac{v_{t,\Delta^c}^i}{v_{t,\Delta^c}^i + v_{t,\delta^c}^i + v_{t,c}^i}, \tag{2}$$

$$\alpha_{t+1}^{i,x} \equiv \frac{v_{t,\Delta^x}^i}{v_{t,\Delta^x}^i + v_{t,\delta^x}^i + v_{t,x}^i}, \tag{3}$$

while we define the *degree of intertemporal consumption positionality* when young and when old, respectively, as

$$\beta_t^{i,c} \equiv \frac{v_{t,\delta^c}^i}{v_{t,\Delta^c}^i + v_{t,\delta^c}^i + v_{t,c}^i}, \tag{4}$$

$$\beta_{t+1}^{i,x} \equiv \frac{v_{t,\delta^x}^i}{v_{t,\Delta^x}^i + v_{t,\delta^x}^i + v_{t,x}^i}. \tag{5}$$

Here, sub-indexes indicate partial derivative, i.e., $v_{t,c}^i \equiv \partial v_t^i(\cdot) / \partial c_t^i$, $v_{t,x}^i \equiv \partial v_t^i(\cdot) / \partial x_{t+1}^i$, $v_{t,\Delta^c}^i \equiv \partial v_t^i(\cdot) / \partial \Delta_t^{i,c}$, $v_{t,\Delta^x}^i \equiv \partial v_t^i(\cdot) / \partial \Delta_{t+1}^{i,x}$, $v_{t,\delta^c}^i \equiv \partial v_t^i(\cdot) / \partial \delta_t^{i,c}$, and $v_{t,\delta^x}^i \equiv \partial v_t^i(\cdot) / \partial \delta_{t+1}^{i,x}$.

Note that Eqs. (2) and (3) reflect comparisons with other people's current consumption, i.e., the keeping-up-with-the-Joneses motive for relative consumption. The variable $\alpha_t^{i,c}$, which denotes the degree of current consumption positionality when young, is interpretable as the fraction of the overall utility increase from an additional dollar spent on private consumption when young in period t that is due to the increased consumption relative to the average consumption in period t . For example, if $\alpha_t^{i,c} = 0.3$, then 30% of the utility increase from the last dollar spent by the individual when young in period t is due to the increased relative consumption compared with other people's current consumption in the same period; hence, 70% is due to a combination of increased absolute consumption and increased relative consumption compared with other people's past consumption. The variable $\alpha_{t+1}^{i,x}$ has an analogous interpretation for the old individual in period $t + 1$. Eqs. (4) and (5) reflect comparisons with other people's past consumption, i.e., the catching-up-with-the-Joneses motive for relative consumption. $\beta_t^{i,c}$ denotes the fraction of the overall utility increase from an additional dollar spent when young in period t that is due to the increased consumption relative to other people's past consumption; $\beta_{t+1}^{i,x}$ has an analogous interpretation for the old consumer in period $t + 1$.

Returning finally to Eq. (1), the state-variable public good is governed by the difference equation

$$G_t = g_t + (1 - \xi)G_{t-1}, \tag{6}$$

where g_t is the addition to the public good in period t , provided by the government, and $0 \leq \xi \leq 1$ is the rate of depreciation. Therefore, the traditional flow-variable public good appears as the special case where $\xi = 1$.

Each individual of any generation t treats the measures of reference consumption, i.e., \bar{c}_{t-1} , \bar{c}_t and \bar{c}_{t+1} , as exogenous during optimization (while these measures are of course endogenous to the government, as will be explained below). Let \hat{v}_t^i

⁸ An alternative is the (slightly less technically convenient) ratio comparison, where the individual's relative consumption is defined by the ratio between the individual's own consumption and the reference measure (e.g., Boskin and Sheshinski, 1978; Layard, 1980; Abel, 2005; Wendner and Goulder, 2008). Mujcic and Frijters (2013) test models based on difference comparisons, ratio comparisons and ordinal rank without being able to discriminate between them. Aronsson and Johansson-Stenman (2013b) derived optimal income taxation rules in a static model based on both difference and ratio comparisons and concluded that the main qualitative insights obtained are unaffected by comparison type. The same applies here: although the choice to focus on difference comparisons instead of ratio comparisons will affect the exact form of the policy rules derived below, it is of no significance for the qualitative insights from our analysis.

⁹ Almost all previous studies on optimal tax and/or expenditure policy under relative consumption concerns assume that individuals compare their own consumption with a measure of average consumption. In a study of optimal taxation, Aronsson and Johansson-Stenman (2010) consider alternative reference measures based on within-generation and upward comparisons, respectively, and find tax policy responses that are qualitatively similar to those that follow if the reference point is based solely on the average consumption. Upward comparisons are also analyzed by Micheletto (2011).

denote the hours of work by an individual of ability-type i in period t , while w_t^i denotes the before-tax wage rate, s_t^i savings, and r_t the market interest rate in period t . Also, let $T_t(\cdot)$ and $\Phi_{t+1}(\cdot)$ denote the payments of labor income and capital income taxes in period t and $t+1$, respectively. The individual budget constraint can then be written as

$$w_t^i l_t^i - T_t(w_t^i l_t^i) - s_t^i = c_t^i, \quad (7)$$

$$s_t^i (1 + r_{t+1}) - \Phi_{t+1}(s_t^i r_{t+1}) = x_{t+1}^i. \quad (8)$$

The individual first-order conditions for work hours and savings are standard. Since these conditions are not used to derive the cost benefit rules for public goods analyzed below, they are presented in the [Appendix](#).

Firm behavior

We model the production sector in a standard way: it consists of identical competitive firms, the number of which is normalized to one for notational convenience, producing a homogenous good under constant returns to scale. This output is used for both public and private consumptions, as described in the section “[The government’s optimization problem](#)”. The production function is written as

$$Y_t = F(L_t^1, L_t^2, K_t), \quad (9)$$

where Y_t denotes the output (national product), while $L_t^i \equiv n_t^i l_t^i$ is the total number of hours of work supplied by ability-type i in period t , and K_t is the capital stock in period t . The representative firm obeys the standard optimality conditions

$$F_{L_i}(L_t^1, L_t^2, K_t) = w_t^i \quad \text{for } i = 1, 2, \quad (10)$$

$$F_K(L_t^1, L_t^2, K_t) = r_t. \quad (11)$$

To simplify the calculations below, we introduce an additional assumption, i.e., that the relative wage rate (often called wage ratio) in period t , $\phi_t = w_t^1/w_t^2 = F_{L_1}/F_{L_2}$, does not depend directly on K_t , which holds for standard constant returns to scale production functions such as the Cobb–Douglas and CES. This means that the intertemporal tradeoff faced by the government will be driven solely by the interest rate.

The government’s optimization problem

We will use an as general social welfare function as possible by assuming that social welfare increases with the utility of any individual type alive in any time period, without saying anything more regarding these relationships. Following [Aronsson and Johansson-Stenman \(2010, 2014b\)](#), we then assume that the government faces a general social welfare function as follows:

$$W = W(n_0^1 U_0^1, n_0^2 U_0^2, n_1^1 U_1^1, n_1^2 U_1^2, \dots), \quad (12)$$

which is increasing in each argument. Since ability is assumed to be private information, the public policy must also satisfy a self-selection constraint. As in most of the literature on redistribution under asymmetric information, we consider a case where the government wants to redistribute from the high-ability to the low-ability type, implying that the self-selection constraint must prevent the high-ability type from acting as a mimicker, i.e.,

$$U_t^2 = u_t^2(c_t^2, z_t^2, x_{t+1}^2, \bar{c}_{t-1}, \bar{c}_t, \bar{c}_{t+1}, G_t, G_{t+1}) \\ \geq u_t^2(c_t^1, 1 - \phi_t l_t^1, x_{t+1}^1, \bar{c}_{t-1}, \bar{c}_t, \bar{c}_{t+1}, G_t, G_{t+1}) = \hat{U}_t^2. \quad (13)$$

The left-hand side of Eq. (13) denotes the utility of the high-ability type, while the right-hand side is the utility of the mimicker (a high-ability type who pretends to be a low-ability type); the time endowment available for work hours and leisure is normalized to one. The variable $\phi_t l_t^1$ is interpretable as the mimicker’s labor supply; since $\phi_t = w_t^1/w_t^2 < 1$, we have $\phi_t l_t^1 < l_t^1$. Note also that Eq. (13) is based on the assumption that all income is observable to the government.

The resource constraint for this economy means that the output is used for private consumption as well as private and public investments, and is written as follows:¹⁰

$$F(L_t^1, L_t^2, K_t) + K_t - \sum_{i=1}^2 [n_t^i c_t^i + n_{t-1}^i x_t^i] - K_{t+1} - g_t = 0. \quad (14)$$

The second-best problem will be formulated as a direct decision problem, i.e., to choose $l_t^1, c_t^1, x_t^1, l_t^2, c_t^2, x_t^2, K_t, g_t$, and G_t for all t to maximize the social welfare function presented in Eq. (12) subject to Eqs. (6), (13), and (14). The government also recognizes that the measures of reference consumption are endogenous as defined by the mean-value formula presented in

¹⁰ For notational simplicity, we do not explicitly model the depreciation of physical capital K . However, the change in the capital stock between time periods can alternatively be interpreted as net investment, meaning that depreciation is accounted for implicitly.

the section “The OLG economy and individual preferences”. The Lagrangian can be written as

$$\begin{aligned} \mathcal{L} = & W(n_0^1 U_0^1, n_0^2 U_0^2, n_1^1 U_1^1, n_1^2 U_1^2, \dots) + \sum_t \lambda_t [U_t^2 - \hat{U}_t^2] \\ & + \sum_t \gamma_t \left[F(L_t^1, L_t^2, K_t) + K_t - \sum_{i=1}^2 [n_t^i c_t^i + n_{t-1}^i x_t^i] - K_{t+1} - g_t \right], \\ & + \sum_t \mu_t [g_t + (1 - \xi)G_{t-1} - G_t] \end{aligned} \tag{15}$$

where λ , γ , and μ are Lagrange multipliers. The first-order conditions are presented in the Appendix. These conditions will now be used to analyze the optimal provision of the public good.

A general rule for public good provision when relative consumption matters

In this section, we present general optimality conditions for the public good provision in a format that facilitates straightforward economic interpretations and comparisons with the benchmark case with no relative consumption concerns. More specifically, the optimal provision rules will be expressed in terms of what we will denote the positionality effect, i.e., the welfare effect associated with changed reference consumption per se.

Since our concern is to characterize policy incentives, we will focus on the “rule issue” throughout the paper by analyzing how the relative consumption concerns affect the policy rule for the state-variable public good. Hence, we will not deal with the “level issue,” i.e., not analyze whether the levels of public goods in equilibrium will be smaller or larger than what they would have been in an economy where people had preferences without relative consumption concerns.¹¹

Let $\hat{u}_t^2 = u_t^2(c_t^1, 1 - \phi_t^1, x_{t+1}^1, \bar{c}_{t-1}, \bar{c}_t, \bar{c}_{t+1}, G_t, G_{t+1})$ denote the utility faced by the mimicker of generation t based on the function $u_t^2(\cdot)$ in Eq. (1). We can then define the marginal rate of substitution between the public good and private consumption for the young and old ability-type i , and for the young and old mimicker, in period t as follows:

$$MRS_{G,c}^{i,t} \equiv \frac{u_{t,G_t}^i}{u_{t,c}^i}, \quad MRS_{G,x}^{i,t} \equiv \frac{u_{t-1,G_t}^i}{u_{t-1,x}^i}, \quad \hat{MRS}_{G,c}^{2,t} \equiv \frac{\hat{u}_{t,G_t}^2}{\hat{u}_{t,c}^2}, \quad \text{and} \quad \hat{MRS}_{G,x}^{2,t} \equiv \frac{\hat{u}_{t-1,G_t}^2}{\hat{u}_{t-1,x}^2}.$$

To shorten the formulas to be derived, we shall also use the short notation

$$MB_{t,G} \equiv \sum_i n_t^i MRS_{G,c}^{i,t} + \sum_i n_{t-1}^i MRS_{G,x}^{i,t} \tag{16}$$

for the sum of the marginal willingness to pay for the public good (measured as the marginal rate of substitution between the public good and private consumption) among those alive in period t . We will also use the notation

$$\Omega_t \equiv \lambda_t \hat{u}_{t,c}^2 [MRS_{G,c}^{1,t} - \hat{MRS}_{G,c}^{2,t}] + \lambda_{t-1} \hat{u}_{t-1,x}^2 [MRS_{G,x}^{1,t} - \hat{MRS}_{G,x}^{2,t}] \tag{17}$$

for the difference in the marginal value attached to the public good between low-ability type and the mimicker (measured both for the young and old) in period t . To facilitate later interpretations, we assume that $MB_{t,G}$ is decreasing in G_t .¹² We are now able to derive the following result:

Proposition 1. *The optimal provision of the public good is characterized by*

$$\sum_{\tau=0}^{\infty} \frac{\gamma_{t+\tau}}{\gamma_t} \left[MB_{t+\tau,G} + \Omega_{t+\tau} - \frac{MB_{t+\tau,G}}{N_{t+\tau} \gamma_{t+\tau}} \frac{\partial \mathcal{L}}{\partial \bar{c}_{t+\tau}} \right] [1 - \xi]^\tau = 1. \tag{18}$$

Following Aronsson and Johansson-Stenman (2010), the partial derivative of the Lagrangian with respect to the reference consumption in period t , i.e., $\partial \mathcal{L} / \partial \bar{c}_t$, will be called the *positionality effect* in period t , and reflects the overall welfare consequences of an increase in \bar{c}_t , holding each individual’s own consumption constant. As such, it is a measure of the “positional externality” of private consumption. While it is reasonable to expect $\partial \mathcal{L} / \partial \bar{c}_t$ to be negative, since for each individual $u_{t,\bar{c}_t}^i < 0$ and $u_{t,\bar{c}_{t-1}}^i < 0$, it is theoretically possible that it is positive due to effects through the self-selection constraint discussed more thoroughly in the following sections.

Before interpreting Proposition 1 further, let us consider the special case where $\xi = 1$. In this case, the state-variable public good is equivalent to an atemporal control (or flow) variable, i.e., $G_t = g_t$, and hence we can simplify Eq. (18) and obtain

¹¹ The distinction between the rule and level issues of public good provision can also be exemplified in the context of comparisons between first- and second-best models. The rule issue then refers to the comparison between the optimal second-best provision rule and the first-best (Samuelson) rule, whereas the level issue addresses whether the optimal level of the public good is larger or smaller in equilibrium in a second-best economy than in a first-best one. The answers to the rule and level issues have often been shown to be quite different, see, e.g., Chang (2000) and Gaube (2000, 2005). Wendner and Goulder (2008) is as far as we know the only paper that has dealt with the level issue of public good provision in a second-best model with relative consumption concerns.

¹² A sufficient – but not necessary – condition for this property to hold is that private and public consumption, if measured in the same period, are weak complements in the utility function.

Corollary 1. *If the public good is a flow variable, so that $\xi = 1$, then the optimal provision of the public good satisfies*

$$MB_{t,G} + \Omega_t - \frac{MB_{t,G}}{N_t \gamma_t} \frac{\partial \mathcal{L}}{\partial \bar{c}_t} = 1. \tag{19}$$

Eq. (19) is analogous to the formula for public provision derived in a static model by Aronsson and Johansson-Stenman (2008). The right-hand side is the direct marginal cost of providing the public good, which is measured as the marginal rate of transformation between the public good and the private consumption good and is normalized to one, whereas the left-hand side is interpretable as the marginal benefit of the public good adjusted for the influences of the self-selection constraint and positional preferences, respectively. With a flow variable public good, the main differences between a static model and the intertemporal model analyzed here are that the self-selection effect and positionality effect relevant for public provision in period t reflect the incentives facing generations t and $t - 1$, as the high-ability type in each of these generations may act as a mimicker in period t .

Let us now return to the case with a state-variable public good, i.e., where $\xi < 1$. Eq. (18) essentially combines the policy rule for a state-variable public good in an OLG model without positional preferences (see Pirttilä and Tuomala, 2001) with an indicator of how the marginal benefit of an incremental public good is modified by relative consumption concerns. Again, the right-hand side is the direct marginal cost of a small increase in the contribution to the public good in period t , which is measured as the marginal rate of transformation between the public good and the private consumption good, whereas the left-hand side measures the marginal benefit of an increase in the contribution to the public good in period t adjusted for the influences of the self-selection constraint and positional preferences, respectively. Note that this measure of adjusted marginal benefit is intertemporal as an increase in g_t , ceteris paribus, affects the utility of each ability-type, as well as the self-selection constraint and the welfare the government attaches to increased reference consumption, in all future periods. The latter means that the marginal benefit of an increment to the public good in period t depends on an intertemporal sum of positionality effects; not just the positionality effect in period t . Therefore, positional concerns affect state-variable and flow-variable public goods differently. This will be described more thoroughly below.

Optimal provision rules with keeping-up-with-the-Joneses preferences

The positionality effect included in Eq. (18) is crucial for our understanding of how the incentives underlying public provision depend on the relative consumption concerns. In this section, we assume that the positional preferences are of the keeping-up-with-the-Joneses type, meaning that each individual derives utility from his/her own current consumption relative to the current average consumption in the economy as a whole, and that each individual makes this comparison both when young and when old. As indicated above, we abstract from catching-up-with-the-Joneses comparisons here; such comparisons are addressed in the section “Optimal provision of the public good under both keeping-up-with-the-Joneses and catching-up-with-the-Joneses preferences”. This simplification means that the variables $\delta_t^{i,c} \equiv c_t^i - \bar{c}_{t-1}$ and $\delta_{t+1}^{i,x} \equiv x_{t+1}^i - \bar{c}_t$ vanish from Eq. (1) and, consequently, that the intertemporal degrees of positionality are equal to zero.

When the preferences are of the keeping-up-with-the-Joneses type, the positionality effect only reflects current degrees of positionality. By using Eqs. (2) and (3) – which represent measures of the current degree of positionality at the individual level – we can define the average degree of current consumption positionality in period t as follows:

$$\bar{\alpha}_t = \sum_i \alpha_t^{i,x} \frac{n_{t-1}^i}{N_t} + \sum_i \alpha_t^{i,c} \frac{n_t^i}{N_t} \in (0, 1). \tag{20a}$$

We also introduce an indicator of the difference in the degree of current consumption positionality between the mimicker and the low-ability type in period t , α_t^d , such that

$$\alpha_t^d \equiv \frac{\lambda_{t-1} \hat{u}_{t-t,x}^2}{\gamma_t N_t} [\hat{\alpha}_t^{2,x} - \alpha_t^{1,x}] + \frac{\lambda_t \hat{u}_{t,c}^2}{\gamma_t N_t} [\hat{\alpha}_t^{2,c} - \alpha_t^{1,c}], \tag{20b}$$

where the $\hat{\cdot}$ denotes “mimicker” (as before) and the superindex d stands for “difference.” Thus, α_t^d reflects an aggregate measure of the positionality differences between the young mimicker and the young low-ability type and between the old mimicker and the old low-ability type, respectively. Consequently, $\alpha_t^d > 0$ (< 0) if the mimicker is always, i.e., both when young and old, more (less) positional than the low-ability type. Following Aronsson and Johansson-Stenman (2010), the positionality effect associated with the keeping-up-with-the-Joneses type of positional preferences can then be written as

$$\frac{\partial \mathcal{L}}{\partial \bar{c}_t} = -N_t \gamma_t \frac{\bar{\alpha}_t - \alpha_t^d}{1 - \bar{\alpha}_t}. \tag{21}$$

Therefore, the overall welfare effect of an increase in the level of reference consumption in period t , ceteris paribus, contains two components. The first is the average degree of current positionality, $\bar{\alpha}_t$, which contributes to decrease the right-hand side of Eq. (21). This negative effect arises because the utility facing each individual of generation t depends negatively on \bar{c}_t via the argument $c_t^i - \bar{c}_t$ in the utility function, and the utility facing each individual of generation $t - 1$ depends negatively on \bar{c}_t via the argument $x_{t+1}^i - \bar{c}_t$. As such, the average degree of current positionality reflects the magnitude of the positional externality. The second component in Eq. (21), α_t^d , appears because the mimicker and the (mimicked) low-ability type

typically differ with respect to the degree of positionality, which the government may exploit to relax the self-selection constraint. If $\alpha_t^d > 0$ (< 0), then increased reference consumption in period t leads to a relaxation (tightening) of the self-selection constraint, as it means that the mimicker is hurt more (less) than the low-ability type. As a consequence, this effect may either counteract ($\alpha_t^d > 0$) or reinforce ($\alpha_t^d < 0$) the negative positional consumption externality.

By substituting Eq. (21) into Eq. (18), we can derive the following result:

Proposition 2. *The optimal provision of the public good based on keeping-up-with-the-Joneses preferences is characterized by*

$$\sum_{\tau=0}^{\infty} \frac{\gamma_{t+\tau}}{\gamma_t} \left[MB_{t+\tau,G} \frac{1-\alpha_{t+\tau}^d}{1-\bar{\alpha}_{t+\tau}} + \Omega_{t+\tau} \right] [1-\xi]^\tau = 1. \quad (22)$$

The interesting aspect of Proposition 2 is that the direct effects of positional concerns are captured by a single multiplier, $(1-\alpha_{t+\tau}^d)/(1-\bar{\alpha}_{t+\tau})$, which is interpretable as the “positionality weight” in period $t+\tau$. The average degree of positionality, $\bar{\alpha}_{t+\tau}$, scales up the aggregate instantaneous marginal benefit, ceteris paribus, and, therefore, contributes to increase the provision of the public good. As explained above, the effect of $\alpha_{t+\tau}^d$ (the measure of differences in the degree of positionality between the mimicker and the low-ability type) can be either positive or negative. If $\alpha_{t+\tau}^d > 0$, this mechanism contributes to scale down the marginal benefit of public consumption in period $t+\tau$. The intuition is, of course, that additional resources spent on private consumption leads to a relaxation of the self-selection constraint in this case (as the mimicker is more positional than the low-ability type). If $\alpha_{t+\tau}^d < 0$, on the other hand, this mechanism works in the opposite direction.

Therefore, a sufficient (not necessary) condition for the positionality weight in period $t+\tau$ to scale up the aggregate instantaneous marginal benefit of the public good in that period is that $\alpha_{t+\tau}^d \leq 0$, for which a sufficient condition in turn is that the low-ability type is always at least as positional as the mimicker. By noting that

$$\frac{\gamma_{t+\tau}}{\gamma_t} = \frac{1}{(1+r_{t+1})(1+r_{t+2})\cdots(1+r_{t+\tau})}$$

we derive the following more general result in the Appendix:

Proposition 3. *Conditional on $MB_{t+\tau,G}$, $\Omega_{t+\tau}$, and $r_{t+\tau}$ for all $\tau \geq 0$, a necessary and sufficient condition for the joint impact of present and future positionality effects to increase the contribution to the public good in period t is that*

$$\sum_{\tau=0}^{\infty} MB_{t+\tau,G} \frac{\bar{\alpha}_{t+\tau} - \alpha_{t+\tau}^d}{1-\bar{\alpha}_{t+\tau}} [1-\xi]^\tau > 0.$$

Hence, a sufficient condition is that the low-ability type is predominantly at least as positional as the mimicker in the sense that

$$\sum_{\tau=0}^{\infty} MB_{t+\tau,G} \frac{\alpha_{t+\tau}^d}{1-\bar{\alpha}_{t+\tau}} [1-\xi]^\tau \leq 0.$$

Note that even though the second condition in Proposition 3 is much stronger than the first, it still does not require the low-ability types to be at least as positional as the mimickers in all periods. Instead, as long as the low-ability type is predominantly at least as positional as the mimicker – which imposes a condition on a weighted average of future differences in the degree of positionality – this is perfectly consistent with the possibility that the mimicker is more positional than the low-ability type during certain periods or intervals of time.

By “the joint impact of present and future positionality effects” we thus mean the effect of including the positionality weight in the provision formula, ceteris paribus. Of course, when comparing this provision rule with the conventional one (i.e., with no impact of positional concerns), it is clear that the individuals’ marginal willingness to pay for the public good, as well as the various Lagrange multipliers, will be different too in the different equilibria. Nevertheless, in the limit case where the joint impact of present and future positionality effects is small, these differences will presumably be small as well. This suggests that the conditions in Proposition 3 may also approximately hold as to when the optimal public good provision level according to Eq. (22) will be higher than the level that would follow (in equilibrium) with the conventional provision rule. Hence, it appears fair to say that for a rather broad set of assumptions, applying the modified provision rule in Eq. (22) tends to imply a higher level of public good provision compared with the conventional provision rule that does not reflect the positionality effect.¹³

Let us next consider conditions for when the second-best adjustments through the impacts on the self-selection constraints, i.e., the effects of the variables Ω_t and α_t^d for all t , vanish from the policy rule for public provision. We have derived the following result:

¹³ Nevertheless, this conclusion should not be confused with the level issue in the sense of comparing the optimal provision levels in equilibrium based on preferences with and without relative consumption elements.

Proposition 4. *If leisure is weakly separable from private and public consumption in the sense that the utility function can be written as $U_t^i = q_t^i(h_t(c_t^i, x_{t+1}^i, \Delta_t^{i,c}, \Delta_{t+1}^{i,x}, G_t, G_{t+1}), z_t^i)$ for all t , then the optimal policy rule for the public good is given by*

$$\sum_{\tau=0}^{\infty} \frac{\gamma_{t+\tau} MB_{t+\tau,G}}{\gamma_t (1 - \bar{\alpha}_{t+\tau})} [1 - \xi]^\tau = 1. \tag{23}$$

Note that while we still allow for type-specific preferences, here the function $h_t(\cdot)$ is common to all consumers of generation t . In the absence of relative consumption concerns (in which case $\bar{\alpha}_t \equiv 0$ for all t), Eq. (23) coincides with a dynamic analog to the standard Samuelson condition. This result is modified here because the policy rule still reflects a desire to correct for positional externalities, which works to increase the marginal benefit of the public good (since $1/(1 - \bar{\alpha}_t) > 1$ for all t by assumption). The intuition behind Proposition 4 is that if leisure is weakly separable from the other goods in the utility function – and with the additional restriction that the function $h_t(\cdot)$ is common for the two ability types – it follows that $\Omega_t = \alpha_t^d = 0$ for all t , i.e., neither the marginal willingness to pay for the public good nor the degree of positionality differs between the mimicker and the low-ability type. Thus, there is no longer an incentive for the government to modify the provision of the public good to relax the self-selection constraint.

Following Aronsson and Johansson-Stenman (2008), it is interesting to consider the role of preference elicitation for the public good. Note first that individual benefits of the public good are so far measured by each individual's marginal willingness to pay for a small increment, ceteris paribus, i.e., while holding everything else fixed, including others' private consumption. In a much discussed evaluation of the so-called contingent valuation method for valuing non-market goods, Arrow et al. (1993) recommended the use of a referendum method as a payment vehicle. This means that people are asked whether they would vote yes or no in a referendum concerning the provision (or increased provision) of the public good combined with a payment for this good, typically a tax or charge that would affect all people. We will here focus on the natural benchmark case where all people would have to pay the same amount, although we briefly discuss another interesting possibility in the concluding section.

In conventional economic models without social comparisons, each individual's marginal willingness to pay for the public good does not depend on whether others' private consumption will change in response to increased public provision. On the other hand, if people are concerned with their relative consumption, it does matter how the marginal willingness to pay is measured.¹⁴ To see this, let us define the marginal rate of substitution between the public good and private consumption at any time, t , conditional on the requirement that $c_t^i - \bar{c}_t$ and $x_{t+1}^i - \bar{x}_{t+1}$ remain constant, which would follow if the willingness to pay question was supplemented with the information that everybody has to pay the same amount as the respondent for an incremental public good.

With reference to Eq. (1), this measure of marginal willingness to pay, conditional on that others would have to pay equally much on the margin, can then be defined as follows:

Definition. *An individual's conditional marginal willingness to pay for the public good when young and old, respectively, is defined by:*

$$CMRS_{G,c}^{i,t} \equiv \frac{v_{t,G}^i}{v_{t,c}^i}, \tag{24a}$$

$$CMRS_{G,x}^{i,t} \equiv \frac{v_{t-1,G}^i}{v_{t-1,x}^i}. \tag{24b}$$

In a way similar to Eq. (16), we may construct an aggregate marginal benefit measure consisting of the sum of all people's (alive in period t) marginal willingness to pay for the public good, conditional on that others will also have to pay equally much on the margin, as follows:

$$CMB_{t,G} \equiv \sum_i n_t^i CMRS_{G,c}^{i,t} + \sum_i n_{t-1}^i CMRS_{G,x}^{i,t}. \tag{25}$$

The question is then how the optimal provision rule will change if expressed as a function of $CMB_{t,c}$ instead of $MB_{t,c}$. By using

$$\Psi_t \equiv \text{cov} \left(\frac{1 - \alpha_t}{1 - \bar{\alpha}_t}, \frac{CMRS_{G,c}^t}{CMRS_{G,c}^t} \right)$$

as a short notation for the (normalized) covariance between the degree of non-positionality, measured by $1 - \alpha_t^i$, and the marginal willingness to pay for the public good, we have

¹⁴ This idea was discussed in a first best setting already by Ng (1987), who noticed that relative consumption concerns tend to reduce each individual's marginal willingness to pay for the public good (if measured with the reference consumption held constant), which, in turn, counteracts the incentive to internalize externalities through increased public provision.

Proposition 5. Based on keeping-up-with-the-Joneses preferences, and expressed in terms of conditional marginal WTPs, the optimal policy rule for public good provision is given by

$$\sum_{\tau=0}^{\infty} \frac{\gamma_{t+\tau}}{\gamma_t} \left[CMB_{t+\tau,G} [1 + \Psi_{t+\tau}] [1 - \alpha_{t+\tau}^d] + \Omega_{t+\tau} \right] [1 - \xi]^{\tau} = 1. \quad (26)$$

Compared with Eq. (22), we can observe two differences (in addition to the replacement of $MB_{t+\tau,G}$ with $CMB_{t+\tau,G}$). First and foremost, the benefit-amplifying factor $1/(1 - \bar{\alpha}_{t+\tau})$ is not part of Eq. (26). The intuition is straightforward. If others' consumption is held constant, each individual's willingness to pay for increased public good provision would be reduced by the fact that his/her relative consumption decreases. However, if each individual's relative consumption is held constant, as in Proposition 5, there is obviously no such effect. Second, Eq. (26) includes a factor $[1 + \Psi_{t+\tau}]$, for which the intuition can be given as follows. If the conditional marginal WTP differs between the types, then those with a higher conditional marginal WTP will obtain a utility increase, while the others may face a utility loss. The utility increase, in monetary terms, will more than outweigh the utility loss if and only if those with a higher conditional marginal WTP are less positional, i.e., if and only if the covariance between the degree of non-positionality and the conditional marginal WTP is positive.

It is finally interesting to analyze whether there is some special case in which the second-best policy rule for the public good reduces to a first-best policy rule. It turns out that there is, and the following result gives sufficient conditions under which an intertemporal analog to the Samuelson rule applies:

Proposition 6. If – in addition to the conditions underlying Proposition 4 – we assume that (i) the degree of positionality is the same for both ability types in all periods, both when young and when old, and (ii) the market interest rate is constant over time, then the optimal provision of the public good, expressed in terms of conditional marginal WTPs, is given by

$$\sum_{\tau=0}^{\infty} \frac{CMB_{t,G}}{(1+r)^{\tau}} [1 - \xi]^{\tau} = 1. \quad (27)$$

Given that the degree of current consumption positionality is the same for everybody (in which case there is no correlation between the marginal willingness to pay and the degree of current consumption positionality at the individual level), a weighted sum over time of instantaneous marginal benefits should equal the marginal cost of an incremental public good. In other words, an intertemporal analog to the traditional Samuelson rule applies. What is less clear, perhaps, is how we should apply this or any other policy rule in practice, as we would need information about future generations' marginal willingness to pay for the public good. However, before taking the discussion about implementation to any greater detail, it is important to know the point of departure.

Note also that in the special case where $\xi = 1$, i.e., the case where the public good is a flow variable, Proposition 6 implies

Corollary 2. In addition to the conditions governing Proposition 6 (except for a constant interest rate which is not needed here), suppose that the public good is a flow variable so that $\xi = 1$. The policy rule for the public good then simplifies to read

$$CMB_{t,G} = 1. \quad (28)$$

Corollary 2 thus implies that the conventional Samuelson (1954) rule, expressed in marginal willingness to pay conditional on the fact that also others have to pay on the margin, holds for each moment in time.

Optimal provision of the public good under both keeping-up-with-the-Joneses and catching-up-with-the-Joneses preferences

The analysis carried out in the previous section is based on the assumption that the only measure of reference consumption at the individual level, in any period, is based on the average consumption in that particular period. Although this idea accords well with earlier literature on public policy and positional preferences, it neglects the possibility that agents also compare their own current consumption with both their own past consumption and that of other people. In this section, we present and analyze the more general model that takes all these comparisons into account.

Note once again that Eq. (18) holds generally, i.e., irrespective of which form the relative consumption concerns take. To be able to consider keeping-up-with-the-Joneses preferences simultaneously with catching-up-with-the-Joneses preferences, we must explore the positionality effect, $\partial \mathcal{L} / \partial \bar{c}_t$, for this more general case. The positionality effect will then depend on both the current and intertemporal degrees of positionality. Therefore, in a way similar to the average degree of current consumption positionality in Eq. (20a), we use Eqs. (4) and (5) to define the average degree of intertemporal consumption positionality as follows:

$$\bar{\beta}_t = \sum_i \beta_t^{i,x} \frac{n_{t-1}^i}{N_t} + \sum_i \beta_t^{i,c} \frac{n_t^i}{N_t} \in (0, 1). \quad (29a)$$

Furthermore, and by analogy to the variable α_t^d defined in Eq. (20b), which is a summary measure of differences in the degree of current positionality between the mimicker and the low-ability type in period t , we define a corresponding

measure of differences in the degree of intertemporal positionality between the mimicker and the low-ability type,

$$\beta_t^d = \frac{\lambda_{t-1} \hat{u}_{t-1,x}^2}{\gamma_t N_t} [\hat{\beta}_t^{2,x} - \beta_t^{1,x}] + \frac{\lambda_t \hat{u}_{t,c}^2}{\gamma_t N_t} [\hat{\beta}_t^{2,c} - \beta_t^{1,c}]. \tag{29b}$$

The variable β_t^d has the same general interpretation as α_t^d . In other words, $\beta_t^d > 0$ (< 0) if the young and old mimickers in period t are more (less) positional than the corresponding low-ability type, where positionality is measured relative to other people’s past consumption. To simplify the notation and facilitate comparison with Eq. (21), we use the following short notation:

$$B_t = \frac{N_t \gamma_t [\alpha_t^d - \bar{\alpha}_t]}{1 - \bar{\alpha}_t} + \frac{N_{t+1} \gamma_{t+1} [\beta_{t+1}^d - \bar{\beta}_{t+1}]}{1 - \bar{\alpha}_t}.$$

We show in the Appendix (along with the proof of Proposition 7 below) that the positionality effect associated with this more general model can be written as

$$\frac{\partial \mathcal{L}}{\partial c_t} = B_t + \sum_{i=1}^{\infty} B_{t+i} \prod_{j=1}^i \frac{\bar{\beta}_{t+j}}{1 - \bar{\alpha}_{t+j-1}}. \tag{30}$$

The variable B_t in Eq. (30) is analogous to the right-hand side of Eq. (21) with the modification that it also reflects the intertemporal (not just the current) degrees of positionality. As such, it contains two additional components. First, $-N_{t+1} \gamma_{t+1} \bar{\beta}_{t+1} / (1 - \bar{\alpha}_t) < 0$ is interpretable as the value of the positional externality associated with the catching-up-with-the-Joneses motive for relative consumption comparisons. The underlying mechanism is, of course, that \bar{c}_t directly affects individual utility negatively via the argument $x_{t+1}^i - \bar{c}_t$ in the utility function. Second, the component $N_{t+1} \gamma_{t+1} \beta_{t+1}^d / (1 - \bar{\alpha}_t)$ reflects the corresponding welfare effects through the self-selection mechanism in period $t+1$. In a way similar to the analogous measure of differences in the current degree of positionality between the mimicker and the low-ability type, this effect means that increased reference consumption in period t may either contribute to relax ($\beta_{t+1}^d > 0$) or tighten ($\beta_{t+1}^d < 0$) the self-selection constraint. The final component on the right-hand side of Eq. (30) arises due to an intertemporal chain reaction: the intuition is that the catching-up-with-the-Joneses motive for consumption comparisons, i.e., that other people’s past consumption affects utility, means that the welfare effects of changes in the reference consumption are no longer time-separable (as they would be without intertemporal consumption comparisons).

By substituting Eq. (30) into Eq. (18), we can derive the following result:

Proposition 7. *The optimal provision of the public good based on keeping-up-with-the-Joneses and catching-up-with-the-Joneses preferences is characterized as*

$$\sum_{\tau=0}^{\infty} \frac{\gamma_{t+\tau}}{\gamma_t} \left[MB_{t+\tau,G} + \Omega_{t+\tau} - \frac{MB_{t+\tau,G}}{N_{t+\tau} \gamma_{t+\tau}} \left[B_t + \sum_{i=1}^{\infty} B_{t+i} \prod_{j=1}^i \frac{\bar{\beta}_{t+j}}{1 - \bar{\alpha}_{t+j-1}} \right] \right] [1 - \xi]^\tau = 1. \tag{31}$$

The basic intuition behind Proposition 7 is analogous to that of Proposition 2, yet with the modification that the catching-up-with-the-Joneses motive for consumption comparisons is present in Eq. (31). This means that (i) increases in the average degrees of positionality (in both the current and intertemporal dimensions) typically contribute to increased provision of the public good, and (ii) differences in the degrees of positionality between the mimicker and the low-ability type contribute to increase (decrease) the optimal provision of the public good if the low-ability type is predominantly more (less) positional than the mimicker in both dimensions, ceteris paribus.

At the same time, Eq. (31) is not very tractable, due to the intertemporal chain reaction caused by the catching-up-with-the-Joneses motive for relative consumption, and the interpretation of its different components is far from obvious. Yet, by making some additional simplifying assumptions, we are able to simplify the positionality effect given by Eq. (30) considerably, and also derive a policy rule for public provision that takes the same form as Eq. (22).

Assumptions A. $\bar{\alpha}_t = \bar{\alpha}$, $\bar{\beta}_t = \bar{\beta}$, $\alpha_t^d = \alpha^d$, $\beta_t^d = \beta^d$, $N_t = N$, and $r_t = r \quad \forall t$

In other words, the population size and interest rate are assumed to be constant over time, as are our measures of average degrees of positionality and positionality differences between the types of people. While these are of course important restrictions, they are hardly very strong assumptions, and similar assumptions are frequently made in the comparable catching-up-with-the-Joneses literature.¹⁵ These assumptions are consistent with, but are not sufficient conditions for, a growth path with a constant growth rate. It should also be noted that the model is still general enough to reflect different preferences between ability-types including different degrees of positionality.

Following Aronsson and Johansson-Stenman (2014b), we can now define the average degree of total consumption positionality and the difference in the degree of total consumption positionality between the mimicker and the low-ability

¹⁵ See, e.g., Campbell and Cochrane (1999) and Díaz et al. (2003).

type, respectively, in present value terms as

$$\bar{\rho} \equiv \bar{\alpha} + \frac{\bar{\beta}}{1+r},$$

$$\rho^d \equiv \alpha^d + \frac{\beta^d}{1+r}.$$

Therefore, the average degree of total consumption positionality is measured as the average degree of current consumption positionality plus the present value of the average degree of intertemporal consumption positionality; the reason for calculating the present value is, of course, that the intertemporal externality caused in period t gives rise to disutility in period $t+1$. We are not aware of any study that simultaneously attempts to estimate the average degree of current and intertemporal consumption positionality, which is an important task for future research. Moreover, although these two measures are distinctly different from one another and in principle possible to estimate independently,¹⁶ one cannot rule out that existing estimates of current consumption positionality partly reflect also intertemporal consumption positionality.

In a similar way, the measure of differences in the degree of total consumption positionality between the mimicker and the low-ability type, ρ^d , reflects differences in the degree of current consumption positionality, α^d , and the (present value of) differences in intertemporal positionality, $\beta^d/(1+r)$. We can then, under [Assumption A](#), show (see [Appendix](#)) that Eq. (30) reduces to

$$\frac{\partial \mathcal{L}}{\partial \bar{c}_t} = N \gamma_t \frac{\rho^d - \bar{\rho}}{1 - \bar{\rho}},$$

which takes the same general form as in the absence of the catching-up-with-the-Joneses motive for relative consumption, i.e., as Eq. (21). This implies that we are able to present straightforward extensions of [Propositions 2–6](#) and [Corollary 2](#) to the more general case, where we also consider catching-up-with-the-Joneses preferences:

Proposition 2'. *Under [Assumption A](#), and based on keeping-up-with-the-Joneses and catching-up-with-the-Joneses preferences, the optimal provision of the public good is characterized as*

$$\sum_{\tau=0}^{\infty} \frac{\gamma_{t+\tau}}{\gamma_t} \left[MB_{t+\tau, G} \frac{1 - \rho^d}{1 - \bar{\rho}} + \Omega_{t+\tau} \right] [1 - \xi]^\tau = 1. \quad (32)$$

Technically, the only difference compared with Eq. (22) is that the positionality weight based on current degrees, $(1 - \alpha^d)/(1 - \bar{\alpha})$, is here replaced with a corresponding positionality weight based on total degrees, $(1 - \rho^d)/(1 - \bar{\rho})$. The intuition is that comparisons with other people's past consumption imply that the (young and old) individuals alive today impose a negative positional externality on the individuals alive in the next period, i.e., the higher the consumption in period t , ceteris paribus, the greater the utility loss due to lower relative consumption in period $t+1$. As such, this intertemporal externality must be considered simultaneously with the (atemporal) externality that affects others today. Due to [Assumption A](#), the striking implication of [Proposition 2'](#) is that the current and intertemporal aspects of consumption positionality affect the incentives for public good provision in *exactly* the same way. Therefore, the following results for when positional concerns lead to increased contributions to the public good are analogous to [Proposition 3](#):

Proposition 3'. *Suppose that the relative consumption concerns are based on both keeping-up and catching-up-with-the-Joneses preferences. Under [Assumption A](#), and conditional on $MB_{t+\tau, G}$ and $\Omega_{t+\tau}$ for all $\tau \geq 0$, a necessary and sufficient condition for the joint impact of present and future positionality effects to increase the contribution to the public good in period t is that $\bar{\rho} - \rho^d > 0$. Hence, a sufficient condition is that the low-ability type is at least as positional as the mimicker in the sense that $\rho^d \leq 0$.*

Again, the result from the section "[Optimal provision rules with keeping-up-with-the-Joneses preferences](#)" carries over with the only modifications that $\bar{\alpha}$ and α^d are replaced with $\bar{\rho}$ and ρ^d , respectively. Let us then consider conditions for when the second-best adjustments through the impacts on the self-selection constraints vanish from the policy rule for public provision. We can derive the following analog to [Proposition 4](#):

Proposition 4'. *Given the conditions underlying [Proposition 3'](#), and if leisure is weakly separable from private and public consumption in the sense that the utility function can be written as $U_t^i = q_t^i(h_t(c_t^i, x_{t+1}^i, \Delta_t^{i,c}, \Delta_{t+1}^{i,x}, \delta_t^{i,c}, \delta_{t+1}^{i,x}, G_t, G_{t+1}), z_t^i)$ for all t , then the optimal provision of the public good is characterized as*

$$\sum_{\tau=0}^{\infty} \frac{\gamma_{t+\tau}}{\gamma_t} \frac{MB_{t+\tau, G}}{1 - \bar{\rho}} [1 - \xi]^\tau = 1. \quad (33)$$

Note that the separability condition in this case also includes the variables reflecting consumption comparisons over time. Consequently, the conditions for when the second-best adjustments through the impacts on the self-selection constraints

¹⁶ One way would be to modify the questionnaire-experimental method developed by [Johansson-Stenman et al. \(2002\)](#) such that the subjects would make choices based on information about levels of their own current consumption, the current average consumption, and past average consumption.

vanish also carry over to this more general case. Note also that $\bar{\rho}$ still remains in Eq. (33); the intuition is, of course, that the government has an incentive to correct for (current and intertemporal) positional externalities, even if it is unable to use the public good as an instrument to relax the self-selection constraint.

As a final concern, let us once again define the marginal willingness to pay for the public good with all relevant measures of relative consumption held constant. What are the implications of adding the catching-up-with-the-Joneses preferences to the model analyzed in the section “Optimal provision rules with keeping-up-with-the-Joneses preferences”?¹⁷ By using

$$\Lambda_t = \text{COV} \left(\frac{1 - \alpha_t - \beta_t}{1 - \bar{\alpha}_t - \bar{\beta}_t}, \frac{\text{CMRS}_{G,c}^t}{\text{CMRS}_{G,c}^t} \right)$$

to denote the (normalized) covariance between the total degree of non-positionality, measured by $1 - \alpha_t - \beta_t$, and the conditional marginal WTP for the public good, we have the following analog to Proposition 5:

Proposition 5’. *The optimal provision of the public good based on the keeping-up-with-the-Joneses and catching-up-with-the-Joneses preferences and Assumption A, and expressed in terms of conditional marginal WTPs, can be characterized as*

$$\sum_{\tau=0}^{\infty} \frac{\gamma_{t+\tau}}{\gamma_t} \left[\text{CMB}_{t+\tau,G} \frac{1 - \bar{\alpha} - \bar{\beta}}{1 - \bar{\rho}} [1 + \Lambda_{t+\tau}] [1 - \rho^d] + \Omega_{t+\tau} \right] [1 - \xi]^\tau = 1. \quad (34)$$

There is one important difference between Propositions 5 and 5’. If all relative consumption concerns are governed by the keeping-up-with-the-Joneses motive, as in Proposition 5, then the average degree of consumption positionality vanishes from the policy rule for the contribution to the public good. In Eq. (26), therefore, there was no incentive to modify the formula for public provision in order to correct for positional externalities. This result no longer applies in Eq. (34), since $(1 - \bar{\alpha} - \bar{\beta}) / (1 - \bar{\rho}) < 1$. Although the marginal WTPs for each generation are measured with all aspects of relative consumption held constant at the individual level, discounting of intertemporal positionality degrees tends to reduce the social cost of increased reference consumption. Consequently, if the relative consumption concerns (or parts thereof) are driven by a catching-up-with-the-Joneses motive, there is an incentive for the government to reduce the contribution to the public good, ceteris paribus, to reach the optimal level of correction for positional externalities. The intuition is that the catching-up-with-the-Joneses type of externality is characterized by a time-lag between cause and effect and must, therefore, be internalized before the welfare loss actually surfaces; e.g., because each individual’s consumption in period t leads to positional externalities in period $t+1$. The social cost of spending one additional dollar on public consumption in period t , relative to spending it in period $t+1$, is given by $\gamma_t / \gamma_{t+1} = 1 + r$. Thus, if the government at any time t plans to internalize a positional externality in period $t+1$, it is more costly to do so if this externality is generated by a catching-up-with-the-Joneses comparison than a keeping-up-with-the-Joneses comparison, which explains why externality correction leads to a smaller contribution to the public good in Eq. (34) than in Eq. (26), ceteris paribus.

For the same reason, adding a catching-up-with-the-Joneses motive for relative consumption also implies that we have to modify the results presented in Proposition 6 and Corollary 2, the analogs of which are presented as follows:

Proposition 6’. *If – in addition to the conditions in Proposition 4’ – we assume that the degree of current and intertemporal positionality, respectively, does not differ among ability types, neither for young nor for old individuals, then the optimal provision of the public good, expressed in terms of conditional marginal WTPs, is given by*

$$\sum_{\tau=0}^{\infty} \frac{\gamma_{t+\tau}}{\gamma_t} \text{CMB}_{t+\tau,G} \frac{1 - \bar{\alpha} - \bar{\beta}}{1 - \bar{\rho}} [1 - \xi]^\tau = 1. \quad (35)$$

Corollary 2’. *If – in addition to the conditions in Proposition 6’ – the public good is a flow variable, so that $\xi = 1$, then the optimal provision of the public good is given by*

$$\text{CMB}_{t,G} \frac{1 - \bar{\alpha} - \bar{\beta}}{1 - \bar{\rho}} = 1. \quad (36)$$

Eq. (36) is interpretable as an analog to the basic Samuelson rule, expressed in terms of marginal willingness to pay conditional on that others will also have to pay the same amount for the public good on the margin. Note also that if the interest rate is small enough, meaning that the scale factor on the left-hand side of Eq. (36) is close to one, then the conventional Samuelson condition will still provide a reasonable rule-of-thumb for public provision. This insight is clearly remarkable, since we simultaneously consider (i) a second-best problem with asymmetric information between the government and the private sector, (ii) distributional concerns, (iii) keeping-up-with-the-Joneses preferences, (iv) catching-up-with-the-Joneses preferences, and (v) internal habit formation. Yet, needless to say, the fact that we are not able to claim

¹⁷ With catching-up-with-the-Joneses preferences, this presupposes that each individual’s marginal willingness to pay is measured conditional on the fact that also the earlier generation pays the same amount at the margin. Therefore, such a payment vehicle is based on a more complex thought experiment than when all relative consumption concerns are driven by the keeping-up-with-the-Joneses comparison.

that any of the assumptions underlying [Corollary 2'](#) are biased in a certain direction does of course not mean that they constitute good approximations of the real world.

Conclusion

The present paper is, as far as we know, the first to consider public good provision in a dynamic second-best economy with asymmetric information under optimal taxation, where people care about relative consumption. The model used is an extension of the standard optimal nonlinear income tax model with two ability types. Our approach recognizes three mechanisms behind the positional concerns: each individual compares his/her current consumption with (i) his/her own past consumption, (ii) other people's current consumption (keeping-up-with-the-Joneses), and (iii) other people's past consumption (catching-up-with-the-Joneses). As such, the present paper has in several respects generalized the literature on optimal public expenditure when relative consumption matters.

For presentational reasons, we began by analyzing the simpler case where the comparison with other people's consumption is limited to their current consumption. This situation enabled us to derive several distinct results with respect to the consequences of positional preferences for the optimal provision of public goods. Clearly, as the public good in our model is a state variable, the effects of positional preferences are more complex than in the static model analyzed by [Aronsson and Johansson-Stenman \(2008\)](#). The reason is that the marginal benefit of an incremental contribution to the public good in period t reflects the present value of all future instantaneous marginal benefits. Consider the case where the differences in the degrees of positionality between consumers are small. Then, if each individual's marginal willingness to pay for the public good is measured by holding the contributions by others constant, it follows that the more positional people are on average now and in the future, *ceteris paribus*, the larger the optimal contributions to the public good through adjustments of the optimal policy rule in response to positional externalities. Yet, this is not the case when a referendum format is used, where people are asked for their marginal willingness to pay conditional on the fact that others will also have to pay for the increased public provision. In the latter case, additional conditions are presented for when a dynamic analog of the conventional [Samuelson \(1954\)](#) rule applies.

Adding the intertemporal aspects of relative consumption comparisons gives a richer structure, as it enables us to distinguish between the current and intertemporal degrees of consumption positionality. Although a catching-up-with-the-Joneses motive for relative consumption gives rise to the same basic policy incentives as those caused by a keeping-up-with-the-Joneses motive, comparisons with others' past consumption make the analysis more complex, as the welfare effects of a change in the reference consumption in period t effectively become dependent on the preferences of all future generations. Still, for the case where the degrees of (current and intertemporal) consumption positionality are constant over time, we derive a set of distinct results for public good provision when the relative consumption concerns are governed by both keeping-up-with-the-Joneses and catching-up-with-the-Joneses preferences. Here, the total degree of consumption positionality plays largely the same general role as the current degree of positionality does when all relative consumption concerns are driven by the keeping-up-with-the-Joneses type of preferences.

Let us now return to the problem of climate change discussed initially. The results are then interpretable to mean that relative consumption concerns, whether in comparison with others' current or previous consumption, do have important implications for the calculations of future costs and benefits of climate change policy, and our contribution is to show how such concerns modify the conventional cost-benefit analysis. However, it is also demonstrated that the cost-benefit analysis of climate change can under certain conditions be based on a conventional dynamic analog to the Samuelson condition, provided that the individuals' marginal willingness to pay for reduced climate emissions is calculated such that each individual's relative consumption (rather than others' consumption) is held constant.¹⁸ Perhaps the latter is the most important finding for applied economists who intend to estimate the benefits of investing to reduce climate change emissions, now and in the future: if each individual's marginal willingness to pay is measured with the relative consumption held constant, there is no need to adjust the cost-benefit rule for positional externalities generated by a keeping-up-with-the-Joneses type of comparison. Indeed, as is demonstrated, the resulting optimal provision rule becomes surprisingly simple for quite reasonable benchmark cases, despite a quite general and complex underlying model.

Still, as with all theoretical models, the results naturally follow from the assumptions. In the present paper, we have consistently assumed that each individual compares his/her consumption level with the average consumption level in society, which is (largely for convenience) the most commonly used assumption in the literature. Yet, there are of course alternatives. For example, if we were to follow [Abel \(2005\)](#) and assume that each individual only compares his/her own consumption with the average consumption of his/her cohort, the exact form of the positionality effect would differ from [Eq. \(30\)](#), although most qualitative results would still prevail.¹⁹ In particular, when a referendum format is used to elicit the marginal willingness to pay, basically the same simplification of the optimal provision rule would occur. The reason is, of course, that an additional public good provision combined with the referendum payment vehicle would still keep the relative consumption constant for all individuals. The same argument applies (and for the same reason) if we instead follow

¹⁸ The many practical problems associated with calculating such costs and benefits for future generations are of course immense, and beyond the scope of this paper.

¹⁹ Readers who would like to explore this path are referred to Section 3.4 in [Aronsson and Johansson-Stenman \(2014b\)](#), who develop such a model but without public goods.

Duesenberry (1949) and Schor (1998) in assuming that people primarily compare their own consumption with that of high-ability individuals (or any combination of people for that matter).²⁰

Yet, one may of course also consider other versions of the referendum payment vehicle itself. As suggested by one of the referees, a natural alternative is to ask each individual about his/her marginal willingness to pay given that other people have to pay the same *fraction* of their income as the respondent. Increased provision of the public good would then imply increased relative consumption for the low-ability type and a corresponding decrease for the high-ability type. With difference comparisons, this payment vehicle would clearly modify the public good provision rule compared with the rules derived above. Yet, if we were to combine this payment vehicle with a utility function where the relative consumption is measured by consumption *ratios* rather than consumption *differences*, then a public good increase combined with such a referendum payment vehicle would again imply that the relative consumption is held fixed for all.

In our view, the problems of identifying optimal public policy responses in a world where people are motivated also by social comparisons are generally under-researched, and there are many important aspects left to explore in future research. Examples include public provision of private goods and optimal taxation in a multi-country setting where people also compare their own consumption levels with those in other countries. It would also be useful to supplement existing studies that largely focus on algebraic solutions with simulations to quantify the importance of relative consumption concerns (based on different types of consumption comparisons) for optimal taxation as well as public good provision.

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Appendix

First-order conditions for consumers

The first-order conditions for work hours and savings can be written as follows, if expressed in terms of the utility formulation $u_t^i(\cdot)$ in Eq. (1),

$$u_{t,c}^i w_t^i [1 - T_t'(w_t^i l_t^i)] - u_{t,z}^i = 0,$$

$$-u_{t,c}^i + u_{t,x}^i [1 + r_{t+1}(1 - \Phi_{t+1}'(s_t^i r_{t+1}))] = 0,$$

where $u_{t,c}^i \equiv \partial u_t^i / \partial c_t^i$, $u_{t,z}^i \equiv \partial u_t^i / \partial z_t^i$ and $u_{t,x}^i \equiv \partial u_t^i / \partial x_{t+1}^i$, and $T_t'(w_t^i l_t^i)$ and $\Phi_{t+1}'(s_t^i r_{t+1})$ are the marginal labor income tax rate and the marginal capital income tax rate, respectively.

First-order conditions of the second-best problem

The first-order conditions for l_t^1 , c_t^1 , x_{t+1}^1 , l_t^2 , c_t^2 , x_{t+1}^2 , K_{t+1} , G_t , and g_t are given by

$$-\frac{\partial W}{\partial (n_t^1 U_t^1)} n_t^1 u_{t,z}^1 + \lambda_t \hat{u}_{t,z}^2 \left[\phi_t + l_t^1 \frac{\partial \phi_t}{\partial l_t^1} \right] + \gamma_t n_t^1 w_t^1 = 0, \quad (A1)$$

$$\frac{\partial W}{\partial (n_t^1 U_t^1)} n_t^1 u_{t,c}^1 - \lambda_t \hat{u}_{t,c}^2 - \gamma_t n_t^1 + \frac{n_t^1}{N_t} \frac{\partial \mathcal{L}}{\partial \bar{c}_t} = 0, \quad (A2)$$

$$\frac{\partial W}{\partial (n_t^1 U_t^1)} n_t^1 u_{t,x}^1 - \lambda_t \hat{u}_{t,x}^2 - \gamma_{t+1} n_t^1 + \frac{n_t^1}{N_{t+1}} \frac{\partial \mathcal{L}}{\partial \bar{c}_{t+1}} = 0, \quad (A3)$$

$$-\left[\frac{\partial W}{\partial (n_t^2 U_t^2)} n_t^2 + \lambda_t \right] u_{t,z}^2 + \lambda_t \hat{u}_{t,z}^2 l_t^1 \frac{\partial \phi_t}{\partial l_t^2} + \gamma_t n_t^2 w_t^2 = 0, \quad (A4)$$

²⁰ However, in this case the optimal provision rule based on unconditional marginal willingness-to-pay measures would have to be modified in a non-trivial way compared with the results presented above. With upward comparisons, the marginal willingness to pay of the low-ability type does not need to be adjusted for positional concerns, since all positional externalities are generated by the high-ability type.

$$\left[\frac{\partial W}{\partial(n_t^2 U_t^2)} n_t^2 + \lambda_t \right] u_{t,c}^2 - \gamma_t n_t^2 + \frac{n_t^2}{N_t} \frac{\partial \mathcal{L}}{\partial \bar{c}_t} = 0, \tag{A5}$$

$$\left[\frac{\partial W}{\partial(n_t^2 U_t^2)} n_t^2 + \lambda_t \right] u_{t,x}^2 - \gamma_{t+1} n_t^2 + \frac{n_t^2}{N_{t+1}} \frac{\partial \mathcal{L}}{\partial \bar{c}_{t+1}} = 0, \tag{A6}$$

$$\gamma_{t+1}(1+r_{t+1}) - \gamma_t = 0, \tag{A7}$$

$$\sum_{i=1}^2 \left[\frac{\partial W}{\partial(n_t^i U_t^i)} n_t^i u_{t,G_i}^i + \frac{\partial W}{\partial(n_{t-1}^i U_{t-1}^i)} n_{t-1}^i u_{t-1,G_i}^i \right] + \lambda_t [u_{t,G_i}^2 - \hat{u}_{t,G_i}^2] + \lambda_{t-1} [u_{t-1,G_i}^2 - \hat{u}_{t-1,G_i}^2] + \mu_{t+1}(1-\xi) - \mu_t = 0 \tag{A8}$$

$$-\gamma_t + \mu_t = 0, \tag{A9}$$

where we have used that $w_t^i = F_{L^i}(L_t^1, L_t^2, K_t)$ for $i=1,2$, and $r_t = F_K(L_t^1, L_t^2, K_t)$ from Eqs. (10) and (11), i.e., from the first-order conditions of the firm.

Proof. of Proposition 1

We start by rewriting Eq. (A8) as

$$\sum_{i=1}^2 \left[\frac{\partial W}{\partial(n_t^i u_t^i)} n_t^i u_{t,c}^i MRS_{G,c}^{i,t} + \frac{\partial W}{\partial(n_{t-1}^i u_{t-1}^i)} n_{t-1}^i u_{t-1,x}^i MRS_{G,x}^{i,t} \right] + \lambda_t [u_{t,G_i}^2 - \hat{u}_{t,G_i}^2] + \lambda_{t-1} [u_{t-1,G_i}^2 - \hat{u}_{t-1,G_i}^2] + \mu_{t+1}(1-\xi) - \mu_t = 0. \tag{A10}$$

Next we rewrite Eqs. (A2), (A3), (A5), and (A6) as

$$\frac{\partial W}{\partial(n_t^1 U_t^1)} n_t^1 u_{t,c}^1 = \lambda_t \hat{u}_{t,c}^2 + \gamma_t n_t^1 - \frac{n_t^1}{N_t} \frac{\partial \mathcal{L}}{\partial \bar{c}_t}, \tag{A11}$$

$$\frac{\partial W}{\partial(n_t^2 U_t^2)} n_t^2 u_{t,c}^2 = -\lambda_t u_{t,c}^2 + \gamma_t n_t^2 - \frac{n_t^2}{N_t} \frac{\partial \mathcal{L}}{\partial \bar{c}_t}, \tag{A12}$$

$$\frac{\partial W}{\partial(n_{t-1}^1 U_{t-1}^1)} n_{t-1}^1 u_{t-1,x}^1 = \lambda_{t-1} \hat{u}_{t-1,x}^2 + \gamma_t n_{t-1}^1 - \frac{n_{t-1}^1}{N_t} \frac{\partial \mathcal{L}}{\partial \bar{c}_t}, \tag{A13}$$

$$\frac{\partial W}{\partial(n_{t-1}^2 U_{t-1}^2)} n_{t-1}^2 u_{t-1,x}^2 = -\lambda_{t-1} u_{t-1,x}^2 + \gamma_t n_{t-1}^2 - \frac{n_{t-1}^2}{N_t} \frac{\partial \mathcal{L}}{\partial \bar{c}_t}. \tag{A14}$$

Substituting Eqs. (A11)–(A14) into Eq. (A10), we obtain

$$\gamma_t \left[n_t^1 MRS_{G,c}^{1,t} + n_t^2 MRS_{G,c}^{2,t} + n_{t-1}^1 MRS_{G,x}^{1,t} + n_{t-1}^2 MRS_{G,x}^{2,t} \right] \left[1 - \frac{N_t}{\gamma_t} \frac{\partial \mathcal{L}}{\partial \bar{c}_t} \right] + \lambda_t \hat{u}_{t,c}^2 [MRS_{G,c}^{1,t} - \hat{M}RS_{G,c}^{2,t}] + \lambda_{t-1} \hat{u}_{t-1,x}^2 [MRS_{G,x}^{1,t} - \hat{M}RS_{G,x}^{2,t}] + \mu_{t+1} [1 - \xi] - \mu_t = 0. \tag{A15}$$

Using the short notation

$$\Theta_t = \gamma_t \left[n_t^1 MRS_{G,c}^{1,t} + n_t^2 MRS_{G,c}^{2,t} + n_{t-1}^1 MRS_{G,x}^{1,t} + n_{t-1}^2 MRS_{G,x}^{2,t} \right] \left[1 - \frac{N_t}{\gamma_t} \frac{\partial \mathcal{L}}{\partial \bar{c}_t} \right] + \lambda_t \hat{u}_{t,c}^2 [MRS_{G,c}^{1,t} - \hat{M}RS_{G,c}^{2,t}] + \lambda_{t-1} \hat{u}_{t-1,x}^2 [MRS_{G,x}^{1,t} - \hat{M}RS_{G,x}^{2,t}],$$

we have that $\mu_t = \Theta_t + \mu_{t+1} [1 - \xi]$ and hence $\mu_{t+1} = \Theta_{t+1} + \mu_{t+2} [1 - \xi]$ so that

$$\begin{aligned} \mu_t &= \Theta_t + [\Theta_{t+1} + \mu_{t+2} [1 - \xi]] [1 - \xi] \\ &= \Theta_t + [\Theta_{t+1} + [\Theta_{t+2} + \mu_{t+3} [1 - \xi]] [1 - \xi]] [1 - \xi] \\ &= \Theta_t + \Theta_{t+1} [1 - \xi] + \Theta_{t+2} [1 - \xi]^2 + \dots = \sum_{\tau=0}^{\infty} \Theta_{t+\tau} [1 - \xi]^\tau \\ &= \sum_{\tau=0}^{\infty} \gamma_{t+\tau} \left[MB_{t+\tau,G} + \Omega_{t+\tau} - \frac{MB_{t+\tau,G}}{N_{t+\tau} \gamma_{t+\tau}} \frac{\partial \mathcal{L}}{\partial \bar{c}_{t+\tau}} \right] [1 - \xi]^\tau, \end{aligned} \tag{A16}$$

where in the last step we have substituted back for Θ_t and used Eqs. (16) and (17), i.e., the definitions of $MB_{t+\tau,G}$ and $\Omega_{t+\tau}$. Using finally that $\mu_t = \gamma_t$ from Eq. (A9), and dividing both sides of Eq. (A16) by γ_t , we obtain Eq. (18).

Proof. of Corollary 1

Since

$$\lim_{\xi \rightarrow 1} [1 - \xi]^\tau = 0 \quad \text{for } \tau > 0 \quad \text{and} \quad \lim_{\xi \rightarrow 1} [1 - \xi]^\tau = 1 \quad \text{for } \tau = 0,$$

Eq. (19) follows immediately from Eq. (18).

Proof. of Proposition 2

We will here explore the positionality effect, and then substitute this into Eq. (19). The derivative of the Lagrangian with respect to \bar{c}_t is given by

$$\begin{aligned} \frac{\partial \mathcal{L}}{\partial \bar{c}_t} &= \sum_{i=1}^2 \frac{\partial W}{\partial(n_{t-1}^i U_{t-1}^i)} n_{t-1}^i u_{t-1, \bar{c}_t}^i + \sum_{i=1}^2 \frac{\partial W}{\partial(n_t^i U_t^i)} n_t^i u_{t, \bar{c}_t}^i \\ &\quad + \lambda_{t-1} [u_{t-1, \bar{c}_t}^2 - \hat{u}_{t-1, \bar{c}_t}^2] + \lambda_t [u_{t, \bar{c}_t}^2 - \hat{u}_{t, \bar{c}_t}^2]. \end{aligned} \tag{A17}$$

From Eq. (1), we have $u_{t,c}^i = v_{t,c}^i + v_{t,\Delta}^i$, $u_{t,\bar{c}_t}^i = -v_{t,\Delta}^i$, $u_{t,x}^i = v_{t,x}^i + v_{t,\Delta_{t+1}}^i$, and $u_{t,\bar{c}_{t+1}}^i = -v_{t,\Delta_{t+1}}^i$, so

$$u_{t,\bar{c}_t}^i = -\alpha_t^{i,c} u_{t,c}^i, \tag{A18}$$

$$u_{t,\bar{c}_{t+1}}^i = -\alpha_{t+1}^{i,x} u_{t,x}^i. \tag{A19}$$

Corresponding expressions hold for the mimicker. By combining Eqs. (A17–A19) and the corresponding expressions for the mimicker, we obtain

$$\begin{aligned} \frac{\partial \mathcal{L}}{\partial \bar{c}_t} &= - \sum_{i=1}^2 \frac{\partial W}{\partial(n_{t-1}^i U_{t-1}^i)} n_{t-1}^i \alpha_t^{i,x} u_{t-1,x}^i - \sum_{i=1}^2 \frac{\partial W}{\partial(n_t^i U_t^i)} n_t^i \alpha_t^{i,c} u_{t,c}^i \\ &\quad - \lambda_{t-1} [\alpha_t^{2,x} u_{t-1,x}^2 - \hat{\alpha}_t^{2,x} \hat{u}_{t-1,x}^2] - \lambda_t [\alpha_t^{2,c} u_{t,c}^2 - \hat{\alpha}_t^{2,c} \hat{u}_{t,c}^2]. \end{aligned} \tag{A20}$$

Substituting Eqs. (A11)–(A14) into Eq. (A20) gives

$$\begin{aligned} \frac{\partial \mathcal{L}}{\partial \bar{c}_t} &= -\frac{\bar{\alpha}_t \gamma_t N_t}{1 - \bar{\alpha}_t} + \frac{1}{1 - \bar{\alpha}_t} [\lambda_{t-1} \hat{u}_{t-1,x}^2 \{\hat{\alpha}_t^{2,x} - \alpha_t^{1,x}\} + \lambda_t \hat{u}_{t,c}^2 \{\hat{\alpha}_t^{2,c} - \alpha_t^{1,c}\}] \\ &= N_t \gamma_t \frac{\alpha_t^d - \bar{\alpha}_t}{1 - \bar{\alpha}_t}, \end{aligned} \tag{A21}$$

where in the last step we have used the definition of α_t^d . Substituting Eq. (A21) into Eq. (18) gives, finally, Eq. (22).

Proof. of Proposition 3

That the joint impact of present and future positionality effects increases the contribution to the public good in period t means that the benefit side is amplified by such effects compared with the optimal provision rule without such concerns, i.e., that

$$\sum_{\tau=0}^{\infty} \frac{\gamma_{t+\tau}}{\gamma_t} \left[MB_{t+\tau,G} \frac{1 - \alpha_{t+\tau}^d}{1 - \bar{\alpha}_{t+\tau}} + \Omega_{t+\tau} \right] [1 - \xi]^\tau > \sum_{\tau=0}^{\infty} \frac{\gamma_{t+\tau}}{\gamma_t} [MB_{t+\tau,G} + \Omega_{t+\tau}] [1 - \xi]^\tau,$$

where all variables are evaluated in the same equilibrium. Hence,

$$\sum_{\tau=0}^{\infty} \frac{\gamma_{t+\tau}}{\gamma_t} MB_{t+\tau,G} \frac{1 - \alpha_{t+\tau}^d}{1 - \bar{\alpha}_{t+\tau}} [1 - \xi]^\tau - \sum_{\tau=0}^{\infty} \frac{\gamma_{t+\tau}}{\gamma_t} MB_{t+\tau,G} [1 - \xi]^\tau > 0,$$

which directly implies the first inequality of Proposition 3. That the second inequality constitutes a sufficient condition for the first one is trivial since $\bar{\alpha}_t > 0$ for all t .

Proof. of Proposition 4

If leisure is weakly separable from private and public consumption as specified for all t , and where the sub-utility function $h_t(\cdot)$ is the same for both ability types, then clearly $\hat{MRS}_{G,c}^{2,t} = MRS_{G,c}^{1,t}$ and $\hat{MRS}_{G,x}^{2,t} = MRS_{G,x}^{1,t}$, implying that $\Omega_t = 0$. Moreover, the positionality degrees will be the same for the mimicker and the low-ability type, implying that $\hat{\alpha}_t^{2,c} = \alpha_t^{1,c}$ and $\hat{\alpha}_t^{2,x} = \alpha_t^{1,x}$, so that $\alpha_t^d = 0$ for all t . Substituting $\alpha_t^d = 0$ and $\Omega_t = 0$ for all t into Eq. (22) implies Eq. (23).

Proof. of Proposition 5

Since $MRS_{G,c}^{i,t} = u_{t,G_t}^i / u_{t,c}^i$, $CMRS_{G,c}^{i,t} = v_{t,G_t}^i / v_{t,c}^i$, and $u_{t,G_t}^i = v_{t,G_t}^i$, it follows that $MRS_{G,c}^{i,t} = [v_{t,c}^i / u_{t,c}^i] CMRS_{G,c}^{i,t}$. Using that $u_{t,c}^i = v_{t,c}^i + v_{t,\Delta_t}^i$ then implies that

$$MRS_{G,c}^{i,t} = \frac{v_{t,c}^i}{v_{t,c}^i + v_{t,\Delta_t}^i} CMRS_{G,c}^{i,t} = (1 - \alpha_t^{i,c}) CMRS_{G,c}^{i,t}. \tag{A22}$$

Similarly, when old we have

$$MRS_{G,x}^{i,t} = (1 - \alpha_t^{i,x}) CMRS_{G,x}^{i,t}. \tag{A23}$$

Substituting Eqs. (A22) and (A23) into Eq. (16) then implies

$$\begin{aligned} MB_{t,G} &= \sum_i n_t^i (1 - \alpha_t^{i,c}) CMRS_{G,c}^{i,t} + \sum_i n_{t-1}^i (1 - \alpha_t^{i,x}) CMRS_{G,x}^{i,t} \\ &= (1 - \bar{\alpha}_t) CMB_{t,G} \left[1 + \text{cov} \left(\frac{1 - \alpha_t}{1 - \bar{\alpha}_t}, \frac{CMRS_{G,c}^{i,t}}{CMRS_{G,c}^{i,t}} \right) \right] \\ &= (1 - \bar{\alpha}_t) CMB_{t,G} [1 + \Psi_t], \end{aligned} \tag{A24}$$

where we have used the previously defined Ψ_t . Substituting Eq. (A24) into Eq. (22) implies Eq. (26).

Proof. of Proposition 6

From the conditions in Proposition 4 it follows that $\alpha_t^i = 0$ and $\Omega_t = 0$. Moreover, since the degree of positionality is the same for both types, we also have that $\Psi_t = 0$. From Eq. (A7) it follows, since the interest rate is constant, that $\gamma_{t+1} / \gamma_t = 1 / (1 + r)$, and hence that $\gamma_{t+\tau} / \gamma_t = 1 / (1 + r)^\tau$. Substituting these conditions into Eq. (26) gives Eq. (27).

Proof. of Corollary 2

Once again, since

$$\lim_{\xi \rightarrow 1} [1 - \xi]^\tau = 0 \quad \text{for } \tau > 0 \quad \text{and} \quad \lim_{\xi \rightarrow 1} [1 - \xi]^\tau = 1 \quad \text{for } \tau = 0,$$

Eq. (28) follows immediately from Eq. (27).

Proof. of Proposition 7

We will first derive the positionality effect in this more general case and then substitute this effect into Eq. (19). The derivative of the Lagrangian with respect to \bar{c}_t can be written as

$$\begin{aligned} \frac{\partial \mathcal{L}}{\partial \bar{c}_t} &= \sum_{i=1}^2 \frac{\partial W}{\partial (n_{t-1}^i U_{t-1}^i)} n_{t-1}^i u_{t-1, \bar{c}_t}^i + \sum_{i=1}^2 \frac{\partial W}{\partial (n_t^i U_t^i)} n_t^i u_{t, \bar{c}_t}^i \\ &+ \sum_{i=1}^2 \frac{\partial W}{\partial (n_{t+1}^i U_{t+1}^i)} n_{t+1}^i u_{t+1, \bar{c}_t}^i + \lambda_{t-1} [u_{t-1, \bar{c}_t}^2 - \hat{u}_{t-1, \bar{c}_t}^2] \\ &+ \lambda_t [u_{t, \bar{c}_t}^2 - \hat{u}_{t, \bar{c}_t}^2] + \lambda_{t+1} [u_{t+1, \bar{c}_t}^2 - \hat{u}_{t+1, \bar{c}_t}^2]. \end{aligned} \tag{A25}$$

From Eq. (1) we have

$$u_{t,c}^i = v_{t,c}^i + v_{t,\Delta_t^c}^i + v_{t,\delta_t^c}^i = \frac{v_{t,\Delta_t^c}^i}{\alpha_t^{i,c}} = \frac{v_{t,\delta_t^c}^i}{\beta_t^{i,c}},$$

$$u_{t,x}^i = v_{t,x}^i + v_{t,\Delta_t^x}^i + v_{t,\delta_t^x}^i = \frac{v_{t,\Delta_t^x}^i}{\alpha_{t+1}^{i,x}} = \frac{v_{t,\delta_t^x}^i}{\beta_{t+1}^{i,x}},$$

$$u_{t,\bar{c}_t}^i = -v_{t,\Delta_t^c}^i - v_{t,\delta_t^c}^i,$$

$$u_{t,\bar{c}_{t-1}}^i = -v_{t,\delta_t^c}^i,$$

$$u_{t,\bar{c}_{t+1}}^i = -v_{t,\Delta_{t+1}^x}^i,$$

so

$$u_{t,\bar{c}_t}^i = -\alpha_t^{i,c} u_{t,c}^i - \beta_{t+1}^{i,x} u_{t,x}^i, \tag{A26}$$

$$u_{t,\bar{c}_{t-1}}^i = -\beta_t^{i,c} u_{t,c}^i, \tag{A27}$$

$$u_{t,\bar{c}_{t+1}}^i = -\alpha_{t+1}^{i,x} u_{t,x}^i, \tag{A28}$$

which substituted into Eq. (A25) imply

$$\begin{aligned} \frac{\partial \mathcal{L}}{\partial \bar{c}_t} = & - \sum_{i=1}^2 \frac{\partial W}{\partial (n_{t-1}^i U_{t-1}^i)} n_{t-1}^i \alpha_{t-1}^{i,x} u_{t-1,x}^i \\ & - \sum_{i=1}^2 \frac{\partial W}{\partial (n_t^i U_t^i)} n_t^i \left[\alpha_t^{i,c} u_{t,c}^i + \beta_{t+1}^{i,x} u_{t,x}^i \right] \\ & - \sum_{i=1}^2 \frac{\partial W}{\partial (n_{t+1}^i U_{t+1}^i)} n_{t+1}^i \beta_{t+1}^{i,c} u_{t+1,c}^i \\ & + \lambda_{t-1} \left[-\alpha_t^{2,x} u_{t-1,x}^2 + \hat{\alpha}_t^{2,x} u_{t-1,x}^2 \right] + \lambda_t \left[-\alpha_t^{2,c} u_{t,c}^2 - \beta_{t+1}^{2,x} u_{t,x}^2 + \hat{\alpha}_t^{2,c} \hat{u}_{t,c}^2 + \hat{\beta}_{t+1}^{2,x} \hat{u}_{t,x}^2 \right] \\ & + \lambda_{t+1} \left[-\beta_{t+1}^{2,c} u_{t+1,c}^2 + \hat{\beta}_{t+1}^{2,c} \hat{u}_{t+1,c}^2 \right]. \end{aligned} \tag{A29}$$

By substituting Eqs. (A11–A14) into Eq. (A29), and collecting terms, we obtain

$$\begin{aligned} \frac{\partial \mathcal{L}}{\partial \bar{c}_t} = & \frac{\partial \mathcal{L}}{\partial \bar{c}_{t+1}} \frac{\bar{\beta}_{t+1}}{1-\bar{\alpha}_t} - N_t \gamma_t \frac{\bar{\alpha}_t}{1-\bar{\alpha}_t} - N_{t+1} \gamma_{t+1} \frac{\bar{\beta}_{t+1}}{1-\bar{\alpha}_t} \\ & + \frac{\lambda_{t-1} \hat{u}_{t-1,x}^2}{1-\bar{\alpha}_t} \left[\hat{\alpha}_t^{2,x} - \alpha_t^{1,x} \right] + \frac{\lambda_t \hat{u}_{t,c}^2}{1-\bar{\alpha}_t} \left[\hat{\alpha}_t^{2,c} - \alpha_t^{1,c} \right] \\ & + \frac{\lambda_t \hat{u}_{t,x}^2}{1-\bar{\alpha}_t} \left[\beta_{t+1}^{2,x} - \beta_{t+1}^{1,x} \right] + \frac{\lambda_{t+1} \hat{u}_{t+1,c}^2}{1-\bar{\alpha}_t} \left[\beta_{t+1}^{2,c} - \beta_{t+1}^{1,c} \right] \\ = & \frac{1}{1-\bar{\alpha}_t} \left[\bar{\beta}_{t+1} \frac{\partial \mathcal{L}}{\partial \bar{c}_{t+1}} + N_t \gamma_t \left[\alpha_t^d - \bar{\alpha}_t \right] + N_{t+1} \gamma_{t+1} \left[\beta_{t+1}^d - \bar{\beta}_{t+1} \right] \right], \end{aligned} \tag{A30}$$

where we have used the short notations α_t^d and β_t^d as defined earlier. Using the definition for B_t and the short notation

$$\varphi_t = \frac{\bar{\beta}_{t+1}}{1-\bar{\alpha}_t},$$

the recursive Eq. (A30) can more conveniently be rewritten as follows:

$$\begin{aligned} \frac{\partial \mathcal{L}}{\partial \bar{c}_t} = & B_t + \varphi_t \frac{\partial \mathcal{L}}{\partial \bar{c}_{t+1}} = B_t + \varphi_t \left[B_{t+1} + \varphi_{t+1} \frac{\partial \mathcal{L}}{\partial \bar{c}_{t+2}} \right] \\ = & B_t + \varphi_t \left[B_{t+1} + \varphi_{t+1} \left[B_{t+2} + \varphi_{t+2} \frac{\partial \mathcal{L}}{\partial \bar{c}_{t+3}} \right] \right] \\ = & B_t + B_{t+1} \varphi_t + B_{t+2} \varphi_t \varphi_{t+1} + B_{t+3} \varphi_t \varphi_{t+1} \varphi_{t+2} \dots \\ = & B_t + \sum_{i=1}^{\infty} B_{t+i} \prod_{j=1}^i \varphi_{t+j-1}. \end{aligned} \tag{A31}$$

Substituting back $\varphi_t = \bar{\beta}_{t+1}/(1-\bar{\alpha}_t)$ into Eq. (A31) implies

$$\frac{\partial \mathcal{L}}{\partial \bar{c}_t} = B_t + \sum_{i=1}^{\infty} B_{t+i} \prod_{j=1}^i \frac{\bar{\beta}_{t+j}}{1-\bar{\alpha}_{t+j-1}}. \tag{A32}$$

Substituting Eq. (A32) into Eq. (18) implies Eq. (31).

Proof. of Proposition 2'

Given Assumption A, Eq. (A32) reduces to the geometric series

$$\begin{aligned} \frac{\partial \mathcal{L}}{\partial \bar{c}_t} = & \frac{N \gamma_t}{1-\bar{\alpha}} \left[\alpha^d - \bar{\alpha} + \frac{\beta^d - \bar{\beta}}{1+r} \right] \sum_{i=0}^{\infty} \left[\frac{\bar{\beta}}{(1-\bar{\alpha})(1+r)} \right]^i \\ = & N \gamma_t \frac{\alpha^d - \bar{\alpha} + (\beta^d - \bar{\beta})/(1+r)}{1-\bar{\alpha} - \bar{\beta}/(1+r)}, \end{aligned} \tag{A33}$$

where in the last step we have implicitly assumed that $0 < \bar{\rho} < (1 - \bar{\alpha})(1 + r)$ so that the series converges. Using the definitions for $\bar{\rho}$ and ρ^d imply further that

$$\frac{\partial \mathcal{L}}{\partial c_t} = N \gamma_t \frac{\rho^d - \bar{\rho}}{1 - \bar{\rho}}, \quad (\text{A34})$$

which substituted into Eq. (18) implies Eq. (32).

Proof. of Proposition 3'

That the joint impact of present and future positionality effects increases the contribution to the public good in period t means that the benefit side is amplified by such effects compared with the optimal provision rule without such concerns, i.e., that

$$\sum_{\tau=0}^{\infty} \frac{\gamma_{t+\tau}}{\gamma_t} \left[MB_{t+\tau,G} \frac{1 - \rho^d}{1 - \bar{\rho}} + \Omega_{t+\tau} \right] [1 - \xi]^\tau > \sum_{\tau=0}^{\infty} \frac{\gamma_{t+\tau}}{\gamma_t} [MB_{t+\tau,G} + \Omega_{t+\tau}] [1 - \xi]^\tau,$$

which is clearly true if and only if $\bar{\rho} - \rho^d > 0$, for which a sufficient condition (remember that $\bar{\rho} > 0$) is that $\rho^d < 0$.

Proof. of Proposition 4'

If leisure is weakly separable from private and public consumption as specified for all t , and where the sub-utility function $h_t(\cdot)$ is the same for both ability types, then clearly $MRS_{G,c}^{2,t} = MRS_{G,c}^{1,t}$ and $MRS_{G,x}^{2,t} = MRS_{G,x}^{1,t}$, implying that $\Omega_t = 0$. Moreover, all positionality degrees will be the same for the mimicker and the low-ability type, implying that $\hat{\alpha}_t^{2,c} = \alpha_t^{1,c}$, $\hat{\alpha}_t^{2,x} = \alpha_t^{1,x}$, $\hat{\beta}_t^{2,c} = \beta_t^{1,c}$, and $\hat{\beta}_t^{2,x} = \beta_t^{1,x}$ for all t , and that $\alpha^d = \beta^d = 0$. Substituting $\alpha^d = 0$, $\beta^d = 0$, and $\Omega_t = 0$ for all t into Eq. (32) implies Eq. (33).

Proof. of Proposition 5'

Combining $MRS_{G,c}^{i,t} = [v_{t,c}^i / u_{t,c}^i] CMRS_{G,c}^{i,t}$ with $u_{t,c}^i = v_{t,c}^i + v_{t,\Delta c}^i + v_{t,\delta c}^i$ implies

$$MRS_{G,c}^{i,t} = \frac{v_{t,c}^i}{v_{t,c}^i + v_{t,\Delta c}^i + v_{t,\delta c}^i} CMRS_{G,c}^{i,t} = (1 - \alpha_t^{i,c} - \beta_t^{i,c}) CMRS_{G,c}^{i,t}. \quad (\text{A35})$$

Similarly, when old we have

$$MRS_{G,x}^{i,t} = (1 - \alpha_t^{i,x} - \beta_t^{i,x}) CMRS_{G,x}^{i,t}. \quad (\text{A36})$$

Substituting Eqs. (A35) and (A36) into Eq. (16) implies

$$\begin{aligned} MB_{t,G} &= \sum_i n_t^i (1 - \alpha_t^{i,c} - \beta_t^{i,c}) CMRS_{G,c}^{i,t} + \sum_i n_{t-1}^i (1 - \alpha_t^{i,x} - \beta_t^{i,x}) CMRS_{G,x}^{i,t} \\ &= (1 - \bar{\alpha}_t - \bar{\beta}_t) CMB_{t,G} \left[1 + \text{cov} \left(\frac{1 - \alpha_t - \beta_t}{1 - \bar{\alpha}_t - \bar{\beta}_t}, \frac{CMRS_{G,c}^t}{CMRS_{G,c}^t} \right) \right] \\ &= (1 - \bar{\alpha}_t - \bar{\beta}_t) CMB_{t,G} [1 + \Lambda_t], \end{aligned} \quad (\text{A37})$$

where we have used the previously defined Λ_t . Substituting Eq. (A37) into Eq. (33) implies Eq. (34).

Proofs. of Proposition 6' and Corollary 2'

Equivalent to the proofs of Proposition 6 and Corollary 2.

References

- Abel, A.B., 1990. Asset prices under habit formation and catching up with the Joneses. *Am. Econ. Rev. Pap. Proc.* 80, 38–42.
- Abel, A.B., 2005. Optimal taxation when consumers have endogenous benchmark levels of consumption. *Rev. Econ. Stud.* 72, 1–19.
- Akerlof, G.A., 1997. Social distance and social decisions. *Econometrica* 65, 1005–1027.
- Alpizar, F., Carlsson, F., Johansson-Stenman, O., 2005. How much do we care about absolute versus relative income and consumption? *J. Econ. Behav. Organ.* 56, 405–421.
- Alvarez-Cuadrado, F., Monteiro, G., Turnovsky, S.J., 2004. Habit formation, catching up with the Joneses, and economic growth. *J. Econ. Growth* 9, 47–80.
- Aronsson, T., Johansson-Stenman, O., 2008. When the Joneses' consumption hurts: optimal public good provision and nonlinear income taxation. *J. Publ. Econ.* 92, 986–997.
- Aronsson, T., Johansson-Stenman, O., 2010. Positional concerns in an OLG model: optimal labor and capital income taxation. *Int. Econ. Rev.* 51, 1071–1095.
- Aronsson, T., Johansson-Stenman, O., 2013a. Conspicuous leisure: optimal income taxation when both relative consumption and relative leisure matter. *Scand. J. Econ.* 115, 155–175.
- Aronsson, T., Johansson-Stenman, O., 2013b. Veblen's theory of the leisure class revisited: implication for optimal income taxation. *Soc. Choice Welf.* 41, 551–578.
- Aronsson, T., Johansson-Stenman, O., 2014a. When Samuelson met Veblen abroad: national and global public good provision when social comparisons matter. *Economica* 81, 224–243.

- Aronsson, T., Johansson-Stenman, O., 2014b. Positional preferences in time and space: optimal income taxation with dynamic social comparisons. *J. Econ. Behav. Organ.* 101, 1–23.
- Arrow, K., Solow, R., Leamer, E., Portney, P., Radner, R., Schuman, H., 1993. Report of the NOAA panel on contingent valuation. 58. *Fed. Regist.* 4601–4614.
- Blanchflower, D.G., Oswald, A.J., 2004. Well-being over time in Britain and the USA. *J. Public Econ.* 88, 1359–1386.
- Boadway, R., Keen, M., 1993. Public goods, self-selection and optimal income taxation. *Int. Econ. Rev.* 34, 463–478.
- Boskin, M.J., Sheshinski, E., 1978. Individual welfare depends upon relative income. *Q. J. Econ.* 92, 589–601.
- Bowles, S., Park, Y.-J., 2005. Inequality emulation, and work hours: was Thorsten Veblen right? *Econ. J.* 115, F397–F413.
- Brekke, K.-A., Howarth, R.B., 2002. Affluence, Well-Being and Environmental Quality. Edward Elgar, Cheltenham.
- Campbell, J.Y., Cochrane, J.H., 1999. By force of habit: a consumption-based explanation of aggregate stock market behavior. *J. Polit. Econ.* 107, 205–251.
- Carlsson, F., Johansson-Stenman, O., Martinsson, P., 2007. Do you enjoy having more than others? Survey evidence of positional goods. *Economica* 74, 586–598.
- Chang, M.C., 2000. Rules and levels in the provision of public goods: the role of complementarities between the public good and taxed commodities. *Int. Tax Publ. Financ.* 7, 83–91.
- Clark, A.E., Frijters, P., Shields, M.S., 2008. Relative income, happiness and utility: an explanation for the Easterlin paradox and other puzzles. *J. Econ. Lit.* 46, 95–144.
- Clark, A., Senik, C., 2010. Who compares to whom? The anatomy of income comparisons in Europe. *Econ. J.* 120, 573–594.
- Corneo, G., Jeanne, O., 1997. Conspicuous consumption, snobbism and conformism. *J. Publ. Econ.* 66, 55–71.
- Corneo, G., Jeanne, O., 1999. Social organization in an endogenous growth model. *Int. Econ. Rev.* 40, 711–725.
- Corazzini, L., Esposito, L., Majorano, F., 2012. Reign in hell or serve in heaven? A cross-country journey into the relative vs absolute perceptions of wellbeing. *J. Econ. Behav. Organ.* 81, 715–730.
- Díaz, A., Pijoan-Mas, J., Ríos-Rull, J.V., 2003. Precautionary savings and wealth distribution under habit formation preferences. *J. Monet. Econ.* 50, 1257–1291.
- Dohmen, T., Falk, A., Fliessbach, K., Sunde, U., Weber, B., 2011. Relative versus absolute income, joy of winning, and gender: brain imaging evidence. *J. Publ. Econ.* 95, 279–285.
- Dupor, B., Liu, W.F., 2003. Jealousy and overconsumption. *Am. Econ. Rev.* 93, 423–428.
- Duesenberry, J.S., 1949. Income, Saving and the Theory of Consumer Behavior. Harvard University Press, Cambridge, MA.
- Easterlin, R.A., 1995. Will raising the incomes of all increase the happiness of all? *J. Econ. Behav. Organ.* 27, 35–47.
- Easterlin, R.A., 2001. Income and happiness: towards a unified theory. *Econ. J.* 111, 465–484.
- Ferrer-i-Carbonell, A., 2005. Income and well-being: an empirical analysis of the comparison income effect. *J. Publ. Econ.* 89, 997–1019.
- Fliessbach, K., Weber, B., Trautner, P., Dohmen, T., Sunde, U., Elger, C.E., Falk, A., 2007. Social comparison affects reward-related brain activity in the human ventral striatum. *Science* 318, 1305–1308.
- Frank, R.H., Hutchens, R.M., 1993. Wages, seniority, and the demand for rising consumption profiles. *J. Econ. Behav. Organ.* 21, 251–276.
- Gaube, T., 2000. When do distortionary taxes reduce the optimal supply of public goods? *J. Publ. Econ.* 76, 151–180.
- Gaube, T., 2005. Financing public goods with income taxation: provision rules vs. provision level. *Int. Tax Publ. Financ.* 12, 319–334.
- Howarth, R.B., 1996. Status effects and environmental externalities. *Ecol. Econ.* 16, 25–34.
- Howarth, R.B., 2006. Optimal environmental taxation under relative consumption effects. *Ecol. Econ.* 58, 209–219.
- Hylland, A., Zeckhauser, R., 1979. Distributional objectives should affect taxes but not program choice or design. *Scand. J. Econ.* 81, 264–284.
- Ireland, N.J., 2001. Optimal income tax in the presence of status effects. *J. Publ. Econ.* 81, 193–212.
- Johansson-Stenman, O., Carlsson, F., Daruvala, D., 2002. Measuring future grandparents' preferences for equality and relative standing. *Econ. J.* 112, 362–383.
- Johansson-Stenman, O., Sterner, T., 2012. Discounting and Relative Consumption. University of Gothenburg, Mimeo.
- Konrad, K., 1992. Wealth seeking reconsidered. *J. Econ. Behav. Organ.* 18, 215–227.
- Layard, R., 1980. Human satisfaction and public policy. *Econ. J.* 90, 737–750.
- Ljungqvist, L., Uhlig, H., 2000. Tax policy and aggregate demand management under catching up with the Joneses. *Am. Econ. Rev.* 90, 356–366.
- Loewenstein, G.F., Sicherman, N., 1991. Do workers prefer increasing wage profiles? *J. Labor Econ.* 9, 67–84.
- Micheletto, L., 2011. Optimal nonlinear redistributive taxation and public good provision in an economy with Veblen effects. *J. Public Econ. Theory* 13, 71–96.
- Milner, A., 2013. On welfare frameworks and catastrophic climate risks. *J. Environ. Econ. Manag.* 65, 310–325.
- Mujcic, R., Frijters, P., 2013. Economic choices and status: measuring preferences for income rank. *Oxf. Econ. Pap.* 65, 47–63.
- Ng, Y.-K., 1987. Relative Income Effects and the Appropriate Level of Public Expenditure. *Oxf. Econ. Pap.*, 39: 293–300.
- Nordhaus, W., 2007. A review of the Stern review on the economics of climate change. *J. Econ. Lit.* 45, 686–702.
- Oswald, A., 1983. Altruism, jealousy and the theory of optimal non-linear taxation. *J. Publ. Econ.* 20, 77–87.
- Pirttilä, J., Tuomala, M., 2001. On optimal non-linear taxation and public good provision in an overlapping generations economy. *J. Publ. Econ.* 79, 485–501.
- Rauscher, M., 1997. Conspicuous consumption, economic growth, and taxation. *J. Econ.* 66, 35–42.
- Rayo, L., Becker, G.S., 2007. Evolutionary efficiency and happiness. *J. Polit. Econ.* 115, 302–337.
- Samuelson, P.A., 1954. The pure theory of public expenditure. *Rev. Econ. Stat.* 36, 387–389.
- Senik, C., 2009. Direct evidence on income comparisons and their welfare effects. *J. Econ. Behav. Organ.* 72, 408–424.
- Schor, J., 1998. *The Overspent American*. Harperperennial, New York.
- Solnick, S., Hemenway, D., 1998. Is more always better?: A survey on positional concerns. *J. Econ. Behav. Organ.* 37, 373–383.
- Solnick, S., Hemenway, D., 2005. Are positional concerns stronger in some domains than in others? *Am. Econ. Rev. Pap. Proc.* 45, 147–151.
- Stern, N.H., 1982. Optimum Taxation with Errors in Administration. *J. Publ. Econ.* 17, 181–211.
- Stern, N.H., 2007. *The Economics of Climate Change: The Stern Review*. Cambridge University Press, Cambridge, UK.
- Stiglitz, J.E., 1982. Self-selection and pareto efficient taxation. *J. Publ. Econ.* 17, 213–240.
- Stutzer, A., 2004. The role of income aspirations in individual happiness. *J. Econ. Behav. Organ.* 54, 89–110.
- Weitzman, M.L., 2009. On modeling and interpreting the economics of catastrophic climate change. *Rev. Econ. Stat.* 91, 1–19.
- Weitzman, M.L., 2010. Risk-adjusted gamma discounting. *J. Environ. Econ. Manag.* 60, 1–13.
- Wendner, R., 2003. Status environmental externality and optimal tax programs. *Econ. Bull.* 8, 1–10.
- Wendner, R., 2005. Frames of reference, the environment and efficient taxation. *Econ. Gov.* 6, 13–31.
- Wendner, R., 2010a. Growth and keeping up with the Joneses. *Macroecon. Dyn.* 14 (S2), 176–199.
- Wendner, R., 2010b. Conspicuous consumption and generation replacement in a model of perpetual youth. *J. Publ. Econ.* 94, 1093–1107.
- Wendner, R., 2014. Ramsey, Pigou, heterogeneous agents, and non-atmospheric consumption externalities. *J. Public Econ. Theory* 16, 491–521.
- Wendner, R., Goulder, L.H., 2008. Status effects, public goods provision, and the excess burden. *J. Publ. Econ.* 92, 1968–1985.