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On the Value of Life in Rich and Poor Countries and Distributional Weights Beyond Utilitarianism

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Abstract. This paper discusses the use of distributional weights in CBA based on a general Bergson-Samuelson SWF. In particular it illustrates some consequences of applying a SWF characterized by constant inequality aversion (which includes classical utilitarianism as a special case), together with a constant relative risk aversion utility function, when calculating the damage costs of global warming. It extends and clarifies earlier unintuitive results, and emphasizes that utility must be seen as fully cardinal in terms of *levels* in this context. In the special case of utilitarianism, on the other hand, it is sufficient to be able to make interpersonal comparisons of utility *changes*.

Key words: cost-benefit analysis, distributional weights, global warming, utility transformations, welfare theory

JEL classification: D61, D62, D63

1. Introduction

The discussion whether to use distributional weights or not in cost-benefit analysis (CBA) is still ongoing,[1] but this issue is beyond the scope of this paper. Instead we will analyze some consequences of applying distributional weights derived from a general Bergson-Samuelson (B-S) social welfare function (SWF). We show that for non-utilitarian SWFs we need richer utility-information where also the absolute value of utility is important. We will then discuss the value of lost lives in rich and poor countries, and in particular derive conditions for when it is theoretically correct to use the same monetary values in these countries.

In practice, distributional weights are not very often applied in CBA, partly because the distributional consequences are often believed to be relatively minor. However, this is obviously not the case for the problem of global warming, where industrialized countries are causing the dominating share of the emissions, whereas the damage costs will largely affect poor countries (Banuri et al. 1996). Con-

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sequently, there has been an intense discussion on how to deal with distributional effects due to global environmental problems, such as the enhanced greenhouse effect. Most studies at present, including the "social cost chapter" (chapter 6) in the Second Assessment Report of the Intergovernmental Panel of Climate Change (IPCC) (Pearce et al. 1996), are based on conventional CBA without distributional weights, generally implying that a lost life in a poor country is associated with a lower monetary cost than a lost life in rich countries. Not surprisingly, this methodology has been questioned (e.g. Ayres and Walter 1991; Ekins 1995) and the recent contributions by Azar and Sterner (1996), Fankhauser et al. (1997) and Azar (1999) all provide cost estimates of the enhanced global warming with regard to distributional concern. This paper will not provide any new monetary estimates, but rather discuss some welfare theoretic points which deserve attention in environmental economics.

Further, although this paper will discuss consequences of applying distributional weights based on varying ethical assumptions, the framework is narrowed to a teleological and welfaristic ethical perspective; see e.g. Sen (1985, 1993) for arguments in favor of a broader welfare concept which does not only include information about utilities. See also Spash (1993), Banuri et al. (1996) and Rayner et al. (1999) for discussions of rights and procedural justice in relation to long-term environmental problems, and Blackorby (1990) and Johansson-Stenman (1998) on the large general problems to go from ethical values to policy conclusions in an imperfect world.

Section 2 discusses the welfare foundation of distributional weights in general, and analyzes under which transformations of the utility (and social welfare) functions the relative welfare weights will remain constant. Section 3 follows Fankhauser et al. (1997)[2] in focusing on a SWF characterized by constant inequality aversion and a constant relative risk aversion (CRRA) utility function. It generalizes some results by Fankhauser et al. and shows that some of their rather unintuitive results (when the relative risk aversion is larger than one) are due to unreasonable implicit assumptions, implying for example that the Pareto principle is violated. Further, it illustrates possible ways to avoid such problems without being restricted to the use of a utilitarian SWF. Section 4 provides a discussion and conclusion.

2. The General Model

2.1. DISTRIBUTIONAL WEIGHTS

Distributional weights are often derived by totally differentiating a conventional Bergson-Samuelson (B-S) SWF $W = w(u_1(y_1), u_2(y_2), \ldots, u_n(y_n))$ as follows (see e.g. Ray 1984):

$$dW = \frac{\partial w}{\partial u_1}\frac{du_1}{dy_1}dy_1 + \ldots + \frac{\partial w}{\partial u_n}\frac{du_n}{dy_n}dy_n = a_1 dy_1 + \ldots + a_n dy_n \tag{1}$$

where a_i is individual i's social marginal utility of consumption (Diamond 1975). These factors can be seen as income depending coefficients to transform individual income changes to changes in social welfare (as long as the changes are small). Conventional public economics theory (see e.g. Atkinson and Stiglitz 1980) gives two reasons for applying distributional weights: (*i*) The SWF may be concave (or quasi-concave) in utilities, so that a utility increase from a low level increases social welfare more than the same utility increase from a high level (or social welfare is higher for the same amount of total utility more evenly distributed). (*ii*) The utilities may be concave in income, so that the same income increase increases utility more for a poor than for a rich individual. In the context of global warming Azar and Sterner (1996) and Azar (1999) focus mainly on reason (*ii*), whereas the calculations in Fankhauser et al. (1997) are based both on reasons (*i*) and (*ii*). However, as recently noted by Azar (1999), there are some pitfalls by using both (*i*) and (*ii*) simultaneously as will be discussed further below.

2.2. TRANSFORMATIONS OF UTILITY AND WELFARE FUNCTIONS

At least since Samuelson's (1947) classic text *Foundations of Economic Analysis* it has been well-known in economics that consumer choices under certainty (in static models) are independent of any positive monotonic transformation of the utility function. However, even at that time it was also clear that such transformations are generally not permissible in welfare analysis where some aggregated utility or welfare measure is used. Consider again a general B-S SWF and a certain set of transformations of the utility functions $v_i(u_i)$, implying a social welfare index $\Psi = w(v_1(u_1(y_1)), \ldots, v_n(u_n(y_n)))$. In order to ensure that the implied economic policy conclusions derived from the maximization of this SWF is independent of the transformations, it is obviously required that the relative welfare weights must be independent of the transformations. Thus, the ratio between the welfare increase associated with a one dollar income increase for different individuals (or countries) a_i/a_j must be independent of the utility transformations made:

$$\frac{\partial w}{\partial v_i}\frac{dv_i}{du_i}\frac{du_i}{dy_i}/\left(\frac{\partial w}{\partial v_j}\frac{dv_j}{du_j}\frac{du_j}{dy_j}\right) = \frac{\partial w}{\partial u_i}\frac{du_i}{dy_i}/\left(\frac{\partial w}{\partial u_j}\frac{du_j}{dy_j}\right) \qquad \forall i, j \tag{2}$$

implying

$$\frac{\partial w}{\partial v_i}\frac{dv_i}{du_i}/\left(\frac{\partial w}{\partial v_j}\frac{dv_j}{du_j}\right) = \frac{\partial w}{\partial u_i}/\frac{\partial w}{\partial u_j} \qquad \forall i, j \tag{3}$$

In the special case of a utilitarian SWF we have $dv_i/du_i = dv_j/du_j$ for all *i, j*, implying that $v_i = A_i + Bu_i$ for all *i*, where A_i and B are constants and where $B > 0$. Hence, (3) holds for any positive *linear* (affine) utility transformation in the utilitarian case.[3] In the general case, however, (3) does not hold and not even a proportional (homogenous) transformation $v_i = Cu_i$ is generally permissible. For (3) to hold in this case we need that

$$\frac{\partial w}{\partial v_i} \Big/ \frac{\partial w}{\partial v_j} = \frac{\partial w}{\partial u_i} \Big/ \frac{\partial w}{\partial u_j} \qquad \forall i, j \tag{4}$$

From the definition of homothetic functions (e.g. Berck and Sydsæter 1993) it follows that (4) holds if and only if w is homothetic in its arguments. Hence, for all homothetic SWFs proportional utility transformations are permissible (in the above sense). Still, for non-homothetic SWFs (4) will not hold. Consider for example the following simple quasi-concave and non-homothetic SWF: $W = u_1 + u_2 + u_1 u_2$, together with the proportional utility transformation $v_i = cu_i$ for $i = 1, 2$. (4) will then not hold with equality and we have instead:

$$\frac{\partial W}{\partial v_1} \Big/ \frac{\partial W}{\partial v_2} = \frac{1 + cu_2}{1 + cu_2} \neq \frac{\partial W}{\partial u_1} \Big/ \frac{\partial W}{\partial u_2} = \frac{1 + u_2}{1 + u_1} \tag{5}$$

In the case where $u_2 > u_1$ simply multiplying both utility functions by 2 (or any constants larger than 1) implies that a relatively larger welfare weight would be given to an income increase for individual 1, and vice versa.[4] Thus, not even information about the utility ratio-scale, which is often considered to be an extremely strong information requirement (see for example Sen 1979, 1997, p. 4), is in the general case sufficient to make welfare evaluations.

It is also immediately clear that any positive monotonic transformation of the SWF, $\Omega = \omega(w)$, would leave the ratio of the welfare weights a_i/a_j (for all i, j) unaffected, since

$$\frac{\partial \omega}{\partial W} \frac{\partial w}{\partial u_i} \frac{du_i}{dy_i} \Big/ \left(\frac{\partial \omega}{\partial W} \frac{\partial w}{\partial u_j} \frac{du_j}{dy_j} \right) = \frac{\partial w}{\partial u_i} \frac{du_i}{dy_i} \Big/ \left(\frac{\partial w}{\partial u_j} \frac{du_j}{dy_j} \right) \qquad \forall i, j \tag{6}$$

Hence, social welfare can be seen as purely ordinal (in a static world without uncertainty), which is a result perfectly analogous to the case of individual consumer choice and positive monotonic transformations of the utility function.

3. The Case of Both Constant Relative Risk Aversion and Constant Inequality Aversion

It is common to assume a special class of utility functions characterized by constant relative risk aversion as proposed by Atkinson (1970):

$$u = \begin{cases} \frac{y^{1-e}}{1-e} + u_0, & e \neq 1 \\ \ln y + u_0, & e = 1 \end{cases} \tag{7}$$

where $e = -yu''/u'$ is the relative risk aversion. $e = 0$ implies a linear utility function, $e = 0$ together with $u_0 = 0$ implies that utility is proportional to income, and $e \to \infty$ corresponds to extreme risk aversion of maximin type. In a similar way, one often assumes constant social inequality aversion in terms of utilities:

$$w = \begin{cases} \sum_{i=1}^{n} \frac{u_i^{1-\gamma}}{1-\gamma}, & \gamma \neq 1 \\ \sum_{i=1}^{n} \ln u_i, & \gamma = 1 \end{cases} \tag{8}$$

where $\gamma = -uw''/w'$ is the social inequality aversion. $\gamma = 0$ implies classical utilitarianism and $\gamma = 1$ is associated with the Bernoulli-Nash SWF, i.e., welfare is the product instead of the sum of individual utilities.[5] $\gamma \to \infty$ implies what is often denoted (somewhat incorrectly) a Rawlsian maxi-min SWF. The SWF is in general not considered possible to observe objectively, but rather to be based on fundamental ethical principles.[6]

3.1. DISTRIBUTIONAL WEIGHTS

By combining (7) and (8), we have that the factors a_i in (1), representing the social marginal utilities of consumption, are given by:

$$a_i = \begin{cases} u_i^{-\gamma} y_i^{-e} = (\frac{y_i^{1-e}}{1-e} + u_0)^{-\gamma} y_i^{-e}, & e \neq 1 \\ u_i^{-\gamma} y_i^{-e} = (\ln y_i + u_0)^{-\gamma} y_i^{-1}, & e = 1 \end{cases} \quad \forall i \tag{9}$$

Note that for $u_0 = 0$, the commonly used special case considered by Fankhauser et al., a_i reduces to:

$$a_i = \begin{cases} \frac{y_i^{-e-\gamma(1-e)}}{(1-e)^{-\gamma}}, & e \neq 1 \\ (\ln y_i)^{-\gamma}/y_i, & e = 1 \end{cases} \quad \forall i \tag{10}$$

We have in this special case that a_i (for all i) is strictly negative for $e > 1$. As noted by Azar (1999), when $e > 1$, γ can be interpreted as a kind of equality aversion instead of an inequality aversion.

Consequently, an income increase for a country (or an individual) will decrease social welfare! This is so even though utility is an increasing function of income, since w becomes a decreasing function in utility. Hence, the explanation for this peculiar implication is that the Pareto principle is violated. Although welfare is often defined quite generally, the assumption that social welfare should be non-decreasing in individual utilities is most often considered to be a minimum requirement for a plausible SWF (e.g. Boadway and Bruce 1984, p. 139).[7] We also see that a_i goes to plus or minus infinity when e goes to 1, depending on whether e is slightly smaller or larger than one, which in turn implies that utility goes to zero from the positive or negative side.[8] This is the main reason for the seemingly somewhat odd results presented by Fankhauser et al. in their Table III, to be discussed further below.

Furthermore, the SWF is not even mathematically defined unless γ is an integer. In general the real-valued mathematical function $A(B) = B^C$, where $B < 0$, is defined only when C is an integer (A is then equal to $(-1)^C(-B)^C$). Consequently, the elements in the SWF given by (8) are mathematically defined for negative utilities only when $1-\gamma$ (and hence γ) is an integer.[9]

Generally, the factors a_i, and hence the distributional weights to be used in CBA, will depend on the utility levels (unless $\gamma = 0$). For example, the welfare weight ratio for a rich and poor country a_r/a_p will in general be far from the same in the case where $u_r = 100$ and $u_p = 50$, say, as in the case where $u_r = 500$ and $u_p = 450$. Thus, applying some kind of weighted utilitarianism with a corresponding social inequality aversion, and hence giving different weights depending on the magnitude of the utilities, makes sense only when the utility level of all countries (or individuals) is positive.[10] Azar (1999) therefore argues that it can be misleading to use this procedure when $e > 1$. But given the more general formulation in (7), we can always find a sufficiently large u_0 so that u is positive for all countries (or individuals), as long as the income of the poorest country is positive.[11]

3.2. EQUAL MONETARY VALUATION PER STATISTICAL LIFE

As noted by Fankhauser et al. (1997), perhaps the most debated issue related to the socio-economic chapter of the IPCC report was the fact that the economic value of a statistical life (VOSL) was lower in poorer countries compared to richer ones. This is a consequence of a positive income elasticity for risk reductions, if we do not apply any distributional weights. Instead, many have argued (for example, Ekins 1995) that one should use the same monetary value for a lost life in rich and poor countries, when aggregating the overall costs for use in a CBA. As a response to this claim, Fankhauser et al. derived the corresponding inequality aversion for when this would be the appropriate thing to do, based on a generalized utilitarian SWF as described above. However, because of the peculiarities mentioned due to zero and negative utility levels, some of their reported results are somewhat misleading, as also noted by Azar (1999). The condition for an equal monetary value of a statistical life (or any other good) to be used in a CBA can be written as follows:

$$\frac{\partial w}{\partial u_r}\frac{du_r}{dy_r}V_r = \frac{\partial w}{\partial u_p}\frac{du_p}{dy_p}V_p \tag{11}$$

or $a_r V_r = a_p V_p$, where V_r and V_p are the VOSL in rich and poor countries, respectively. Alternatively, we may write:

$$\frac{\partial w}{\partial u_p}\Big/\frac{\partial w}{\partial u_r} = \frac{V_r}{V_p}\frac{du_r}{dy_r}\Big/\frac{du_p}{dy_p} \tag{12}$$

or in the case of a functional form given by (7) and (8):

$$\left(\frac{u_r}{u_p}\right)^{\gamma} = \frac{V_r}{V_p}\left(\frac{u_p}{u_r}\right)^{e} = \left(\frac{y_r}{y_p}\right)^{\varepsilon}\left(\frac{u_r}{u_p}\right)^{-e} \tag{13}$$

where ε is the income elasticity (which is assumed constant in the interval) of the VOSL. Solving for γ gives:

$$\gamma = \begin{cases} \varepsilon \frac{\ln\left(\frac{y_r}{y_p}\right)}{\ln\left(\frac{u_r}{u_p}\right)} - e = \frac{\varepsilon \ln\left(\frac{y_r}{y_p}\right)}{\ln\left(\frac{y_r^{1-e}+u_0(1-e)}{y_p^{1-e}+u_0(1-e)}\right)} - e, & e \neq 1 \\ \varepsilon \frac{\ln\left(\frac{y_r}{y_p}\right)}{\ln\left(\frac{u_r}{u_p}\right)} - 1 = \frac{\varepsilon \ln\left(\frac{y_r}{y_p}\right)}{\ln\left(\frac{\ln y_r + u_0}{\ln y_p + u_0}\right)} - 1, & e = 1 \end{cases} \tag{14}$$

For the special case when $u_0 = 0$ discussed by Fankhauser et al. we have:

$$\gamma = \begin{cases} \frac{\varepsilon}{1-e} - e, & e \neq 1 \\ \varepsilon \frac{\ln\left(\frac{y_r}{y_p}\right)}{\ln\left(\frac{\ln y_r}{\ln y_p}\right)} - 1, & e = 1 \end{cases} \tag{15}$$

In this special case we see directly that γ will go to plus or minus infinity when e goes to 1, but also note that γ will be finite for $e = 1$. We can then calculate numerical values for γ as a function of e, ε and u_0. There are of course an infinite number of possible alternatives for u_0, but the purpose here is simply to illustrate the difference when u_0 is shown in order for the utility level not to be negative, and the ratio u_r/u_p to be reasonable. One straight forward way is then to define u_0 implicitly by requiring that

$$\frac{u_r(y_r, u_0)}{u_p(y_p, u_0)} = \frac{u_r(y_r, u^*)}{u_p(y_p, u^*)} = \alpha \tag{16}$$

where α is a constant. Hence, in addition to assuming e and γ, we need to specify the utility ratio between rich and poor countries in order to calculate the appropriate distributional weights. The utilities will always be positive as long as $y_r > y_p$ and $\alpha > 1$, and (14) would then reduce to:

$$\gamma = \varepsilon \frac{\ln\left(\frac{y_r}{y_p}\right)}{\ln(\alpha)} - e, \quad \forall e \tag{17}$$

For example, one may say that the utility of the rich country is 20% larger than that of the poor country. In Table I, we make this assumption so that $\alpha = 1.2$. In a situation with many countries, u_0 is uniquely defined by assuming the utility ratio between two countries, e.g. the richest and the poorest one.

Alternatively, one may argue that it would be more reasonable, when comparing the effects of different assumption about e, if the utility ratio would be an increasing function of e, which is obtained if u_0 is implicitly defined as follows:

$$\frac{u_r(y_r, u_0)}{u_p(y_p, u_0)} = \frac{u_r(y_r, u^{**})}{u_p(y_p, u^{**})} = \beta \left(\frac{y_r}{y_p}\right)^{1-\frac{e}{1+e}} \tag{18}$$

Table I. Calculation of social inequality aversion γ in order to obtain equal monetary valuation per statistical life in rich and poor countries

	Income el. $\varepsilon = 0.35$			Income el. $\varepsilon = 0.66$			Income el. $\varepsilon = 1.00$			Income el. $\varepsilon = 1.20$		
	$u_0 = 0$	$u_0 = u^*$	$u_0 = u^{**}$	$u_0 = 0$	$u_0 = u^*$	$u_0 = u^{**}$	$u_0 = 0$	$u_0 = u^*$	$u_0 = u^{**}$	$u_0 = 0$	$u_0 = u^*$	$u_0 = u^{**}$
$e = 0$	0.35	2.66	0.35	0.66	5.02	0.66	1	7.6	1	1.2	9.12	1.2
$e = 0.5$	–0.3	–1.14	–0.22	0.32	1.22	0.24	1	3.8	0.75	1.4	5.32	1.05
$e = 0.99$	–64	–4.87	–1.27	–33	–2.51	–0.66	1	0.08	0.02	21	1.6	0.42
$e = 1$	–4.93	–4.94	–1.3	–2.58	–2.59	–0.68	0	0	0	1.52	1.52	0.4
$e = 1.01$	66	–5.02	–1.33	35	–2.66	–0.7	1	–0.1	0	–19	1.44	0.38
$e = 1.5$	2.3	–8.74	–2.88	1.68	–6.39	–2.1	1	–3.8	–1.25	0.6	–2.28	–0.75

Notes: Calculations are made for different assumptions on income elasticity of the VOSL ϵ, relative risk aversion e, and constant term in the utility function u_0. Income in rich countries is 4 times the income in poor countries. Income in poor countries is arbitrarily chosen to 1000 income units. The only results which are scale dependent, and hence affected by this assumption, are when $e = 1$ and $u_0 = 0$.

The utility ratio will always be larger than one, provided that $y_r > y_p$, $e > 0$, and $\beta > 0$, and the utilities will still be positive. We can then simplify (14) for the special case when $\beta = 1$:

$$\gamma = \varepsilon(1 + e) - e = e(\varepsilon - 1) + \varepsilon, \quad \forall e \tag{19}$$

In Table I, we calculate γ from (15), (17) and (19) for $u_0 = 0$, $u_0 = u^*$, and $u_0 = u^{**}$, respectively, where we follow Fankhauser et al. in assuming that the income in rich countries is four times that in poor countries, and where the values for $u_0 = 0$ correspond to the values in Table III in Fankhauser et al.

It follows that the peculiar results from Fankhauser et al., that γ will go to +/– infinity for *e* close to one, will not result when u_0 is chosen in a way so as to keep utility positive, and the utility ratio reasonable, in the relevant range. Instead, from (17) and (19) we have now that γ is continuous in *e* (also when $e = 1$). Further, as long as utility is positive, γ can be interpreted as some kind of inequality aversion measure, also when $e > 1$. We also typically have that, in order to keep an equal monetary valuation, the inequality aversion γ increases when the relative risk aversion *e* decreases (and vice versa), and that a higher inequality aversion is needed for larger ε which is intuitive (for $\varepsilon = 0$, no weight factor is needed).

Consequently, the following statement by Fankhauser et al. (1997, p. 261) is not fully correct, and might be misleading: "There are also cases where the notion of common per-unit values would seem untenable [...] there are parameter combinations for which common per-unit values would imply negative values for γ, that is, 'inequality attraction', which could in the limit (when *e* goes to 1) go to a maximax welfare concept ($\gamma = -\infty$). [...] The restriction of equal values then favors the rich." There is no problem *per se* to simultaneously use a SWF characterized by constant inequality aversion and a CRRA utility function where *e* is large. Rather, if the parameter values (in particular u_0) are chosen so that the (cardinal) utility levels and/or the ratio between different utility levels are unreasonable, then, of course, the results will correspondingly be of little use. Further, as mentioned, when $e > 1$ and $u_0 = 0$ utility is negative, the Pareto criterion is violated, and the SWF is in general not even mathematically defined. Hence, it would be incorrect to say that the corresponding numbers in Table I (and hence their Table III) are based on consistent social welfare maximization.

We have also seen that to avoid these problems we do not need to have some *a priori* opinion about a reasonable level of u_0 *per se*, which indeed seems genuinely difficult to have. Instead it is sufficient to be able to make utility ratio-scale measures (between rich and poor countries) in addition to assumptions of *e* and γ.[12] Still, such a ratio-scale measure of utility, e.g. the possibility to say that utility in the rich country is 50% larger than utility in the poor country, demands of course in itself a great deal of information.

4. Conclusion

In this paper we have discussed the consequences and possible traps of applying distributional weights based on a general B-S SWF together with concave utility functions. We have seen that it is possible to apply both a high (close to or larger than 1) parameter of relative risk aversion and a positive inequality aversion parameter without any counterintuitive results, contrary to earlier studies. With the specific functional form considered, this is obtained by making an additional assumption regarding the utility ratio between the rich and the poor country, implicitly implying that a constant term (intercept) in the utility function is quantified. Hence, we have illustrated the need for interpreting utility in a fully cardinal way in terms of levels; interpersonal comparisons of *changes* in utility are thus not sufficient. Given this, there is nothing problematic *per se*, which would cause strange or unintuitive results, by using distributional weights which are based both on quasi-concavity of the welfare function and concave utility functions. This point is important not only in the case of global warming, but for welfare analysis (e.g. of environmental problems) with regard to distributional effects in general.

However, this conclusion implies no general recommendation that one should necessarily apply such distributional weights. Instead, given the use of *some* kind of distributional weights, to vary one parameter instead of at least 2 is of course easier in practice, and the results would certainly be easier to see through for both analysts and policy makers. For practical use in CBA this would then support the application of distributional weights based on a simple utilitarian SWF, combined with concave utility functions. Still, if this analytically less complicated strategy is chosen, together with ethical values which would justify an inequality averse (in utilities) SWF, the interpretation of the parameter used to reflect equity concern would now not solely reflect the degree to which the marginal utility of income decreases in income, or risk aversion. Instead this parameter would reflect a mixture of risk aversion and ethical values, and hence not be observable by actual market behavior (e.g. under risk).

Finally, using distributional weights in CBA is of course no Panacea. Instead, since a major purpose of CBA (some would say *the* major purpose) is to provide policy makers with relevant information for them to be able to make wise decisions, it is still often advisable to also present the result of a conventional unweighted CBA, as well as the actual distribution of the different components (both in physical and monetized form, and over time and spatially) in an as accessible form as possible.

Notes

1. See Harberger (1971, 1977), Hylland and Zeckhauser (1979), Christiansen (1981), Brent (1984), Drèze and Stern (1987), Boadway and Keen (1993), Drèze (1998), and Johansson-Stenman (1999).
2. Much of this analysis is repeated in Fankhauser et al. (1999).

3. Note that the intercepts need not be identical for all individuals.
4. Consider the welfare weight ratio with a utility transformation $a_1/a_2 = (c+c^2u_2)/(c+c^2u_1) = (1 + 2u_2)/(1 + 2u_1)$, and without $a_1/a_2 = (1 + u_2)/(1 + u_1)$. The difference between these ratios with and without the transformation is then given by $(u_2 - u_1)/((1+2u_1)(1+u_1))$ which is clearly larger than zero for $u_2 > u_1$.
5. A monotonic transformation W of the welfare function w can then be written: $W = e^w = e^{\sum_{i=1}^{n} \ln u_i} = \Pi_{i=1}^{n} u_i$. Hence, the welfare function W is given by the product of individual utilities. Note again that welfare (but not utility) is purely ordinal in the sense that the maximization of w is equivalent (in terms of policy) to the maximization of any monotonic increasing transformation of w (such as W).
6. It is sometimes incorrectly argued that SWFs for this reason are arbitrary. But, of course, the fact that something is not objectively observable, such as the ethical rule that one should not steal, does not make the rule to be arbitrary.
7. A SWF which is a function only of utilities and which satisfies the Pareto criterion is denoted Pareto-inclusive welfarism by Sen (1979).
8. But note that for $e = 1$, a will be finite and equal to $\frac{(\ln y)^{-\gamma}}{y}$.
9. A SWF is normally defined (or at least interpreted) as real-valued, since it is hard to give a reasonable interpretation of welfare as a complex number with an imaginary part.
10. This is of course not only valid with a CRRA utility function. Consider the alternative frequently used utility function characterized by constant *absolute* level of risk aversion (CARA): $u = -e^{-\sigma y}$, where the absolute risk aversion $\sigma = -u''/u'$. Utility is then always negative implying that $a < 0$. Here too, this problem could be avoided by adding a sufficiently large constant term to the utility function, i.e., $u = -e^{-\sigma y} + u_0$, so that utility is always positive in the relevant range.
11. The constant term in the utility function is of no use in the case of classical utilitarianism ($\gamma = 0$) if the population size is (approximately) fixed, as it is here. However, in case of varying population it is crucial also for classical utilitarianism; see for example Dasgupta (1998).
12. As we have seen in Section 2, this is because the SWF used is homothetic.

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