Distributional Weights in Cost-Benefit Analysis—Should We Forget about Them?

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ABSTRACT. Many argue that it is socially inefficient to use distributional weights in cost-benefit analysis, and that doing so implies large inefficiency losses, when distributional matters can be dealt with through income taxation, instead. Our results question this view, by showing a large range of cases when distributional weights are (second-best) optimal to use. One example is when different provided goods affect tax-revenues equally per dollar spent; utility functions that are separable in the provided goods is sufficient for this. Most results hold for linear and non-linear income taxes and whether they are optimal or not. General policy implications are discussed. (JEL D61, H21)

I. INTRODUCTION

Cost-benefit analysis (CBA) constitutes a direct link from economic analysis to recommendations for political decisionmakers. Despite this, or perhaps because of this, CBA remains highly controversial. One of the most important reasons is the common practice of focusing solely on efficiency, by simply comparing aggregate maximum willingness to pay (WTP) figures with costs. Indeed, from the political debate, it is clear that distributional matters are often extremely important, especially concerning environmental issues.

However, the current efficiency-only practice is in no way self-evident. Indeed, Alfred Marshall in his inaugural lecture at Cambridge argued that “taking account of the fact that the same sum of money measures a greater pleasure for the poor than for the rich,” when computing benefits of various changes is a “task [that] most properly belongs to the economic organ.” (Marshall 1885, 31). Moreover, distributional concern as reflected by distributional weights were a rather central component of the project appraisal manuals of the seventies, such as Dasgupta, Marglin, and Sen (1972), Little and Mirrlees (1974) and Squire and van der Tak (1975), even though they became gradually less fashionable over time as described by Little and Mirrlees (1991). This is, for example, reflected by the current practice of the World Bank, which in most cases does not use any explicit distributional weights in its CBA. Similarly, recent advanced textbooks in environmental economics, such as Hanley, Shogren, and White (1997) and Freeman (2003), focus very little on distributional concerns, at least within generations. Despite this, it is still not difficult to find proponents of distributional weighted CBA; see, for example, Drèze and Stern (1987) and Dréze (1998). It is also worth noting that the main financial decisionmaking body in the United Kingdom, H.M. Treasury, has recently, for the first time officially endorsed distributional weights in CBA; the details can be found in their Green Book (H.M. Treasury 2003).

The rationale behind distributional weights are typically based on differences in the social marginal utility of income, that is, the (assumed) fact that one dollar to the poor increases social welfare more than one dollar does to the rich. To illustrate

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1 See e.g., Bromley (1990, 2004) and Johansson-Stenman (1998) for discussions of other potential problems of deriving policy conclusions based on CBA.

2 On the other hand, since poverty measures are often used to evaluate policy reforms, this can be seen to reflect some kinds of implicit weights.
this, consider the frequently used assumptions of a utilitarian social welfare function (SWF) in a society with \( n \) individuals, so that \( W = U^1 + U^2 + \ldots + U^n \), and that each individual has a utility function characterized by constant relative risk aversion, so that the utility function can be written as \( U = y^{1-\sigma}/(1 - \sigma) \), where \( \sigma \) is the Arrow-Pratt parameter of relative risk aversion.\(^3\) In social welfare analysis, \( \sigma \) is often seen to reflect social inequality aversion as well; see, for example, Christiansen and Jansen (1978). This implies that the (social) marginal utility of income is given by \( U_x = y^{-\sigma} \), and hence, that the monetary benefit for a poor person in the welfare analysis should be given the relative weight of \( (y_{\text{rich}}/y_{\text{poor}})^\sigma \) compared to the benefit to the rich. According to Dasgupta (1998), empirical evidence based on choice under uncertainty suggests that \( \sigma \) is around 2 on average, or slightly larger, which is also consistent with questionnaire-experimental evidence by Johansson-Stenman, Carlsson, and Daruvala (2002). Blanchard and Fischer (1989, 44) report that results based on intertemporal choices vary greatly, but are often around or larger than unity. If we assume that \( \sigma = 2 \), then the benefit to a person who is 100 times richer than another person should be given a weight of only 1/100\(^2\), that is, 1/10,000, relative to the poor. Obviously, the outcome of the CBA can then be very different with and without weights, for projects that particularly benefit either the rich or the poor.\(^4\)

However, given that equity aspects are intrinsically important, it is still not obvious that one should apply distributional weights in CBA. One may instead, following Harberger (1978), argue that it is more efficient to focus solely on efficiency in CBA and leave distributional considerations to income taxation. A seminal theoretical contribution is due to Hylland and Zeckhauser (1979), authors of “Distributional Objectives Should Affect Taxes But Not Program Choice or Design.” They showed that under optimal non-linear taxation and a specific utility formulation (identical for all individuals), which is weakly separable between private and public goods on one hand and leisure on the other, it is indeed optimal to use the basic Samuelson (1954) rule saying that the sum of individual marginal rates of substitution equals the marginal rate of transformation (\( EMRS = MRT \)) between the publicly provided good and a numeraire (such as private income). Hence, given what may seem to be a reasonable preference structure, it is optimal not to adjust for distributional concerns (and not for incentive effects either); Christiansen (1981) and Broadway and Keen (1993) have further generalized this result.\(^5\) Moreover, using a utility formulation similar to Hylland and Zeckhauser, Shavell (1981) derived conditions when no distributional concern should be taken in legal rule-making, in a model where people choose their own amount of care to avoid accidents.\(^6\) Another way to express these results is that,

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\(^3\) A classical utilitarian framework appears to be the most frequently used in the literature. See Yitzhaki (2003) for an alternative approach, where he derived distributional weights from a SWF which depends directly on inequality indices, such as the Gini coefficient.

\(^4\) Somewhat similarly, Medin, Nyborg, and Bateman (2001) demonstrate that if one instead of assuming equal marginal utility of income for all (as in conventional unweighted CBA) assumes equal marginal utility of the public good, then the aggregate monetary benefit estimates may change substantially.

\(^5\) Hylland and Zeckhauser (1979) showed that a sufficient condition, in addition to optimal non-linear income taxes, for the Samuelson (1954) rule to be applied is that the (identical for all) utility function is separable as follows: \( u(f(x + B(x,G),l)) \), where \( x \) and \( G \) are private and publicly provided consumption, respectively, \( l \) is leisure, and \( B \) is the monetary benefit derived from \( G \). Christiansen (1981) showed that it is sufficient that utility can be written as \( u(f(x,G),l) \). Broadway and Keen (1993), in turn, generalized the result further by showing that it is sufficient that the utility function for any individual \( i \) may be written \( U^i = u^i(f(x^i,G),l^i) \). Hence, it is sufficient that all individuals have an identical subutility function \( f \); the overall utility function \( u \) need not be identical for the Samuelson rule to be valid.

\(^6\) Sanchirico (2000) is a rare exception in the literature. He analyzed a model of legal rule-making in the tradition of Shavell (1981) and, in that specific setting, made very strong conclusions claiming that it is always optimal to use distributional weights. However, see also the comment by Kaplow and Shavell (2000), who claimed that several conclusions by Sanchirico (2000) were unfounded, as well as the rejoinder (in another journal) by Sanchirico (2001).
for a given degree of inequality, the overall economic pie in terms of social welfare is larger if distributional concerns are dealt with solely through income taxation, compared to a situation where they are also dealt with through public provisions of goods, given the underlying assumptions.

While most of this theoretical literature rarely discusses policy implications outside the specific assumptions made, recently a number of papers and textbooks, including Kaplow and Shavell (1994, 2000), Kaplow (1996, 2004), Frank (2000), Ng (2000a, 2000b), and Frank and Bernanke (2001), have argued more generally against the use of distributional weights in CBA as well as in legal decision-making, based on the premise that such weights make the economic pie smaller. The underlying fundamental theoretical arguments in these contributions can be traced back to the Hylland and Zeckhauser (1979) and Christiansen (1981) papers. A crucial issue, however, is whether we really can say that the economic pie would typically be smaller if we used distributional weights in CBA. As will be shown, we cannot generally do that, and sometimes the pie will even be larger with such weights.

From a policy perspective it is, of course, not sufficient to find clear results under special assumptions, which one typically knows are not strictly fulfilled anyway, but rather aim at gaining information about roughly how valid the results are in the real world. For example, if the separability assumptions do not hold, can we then say anything general about distributional concern in CBA? Or, in the example above, can we say, given that we can use income taxation to deal with equity, that the theoretically appropriate weights given to rich and poor people would most likely not differ by more than, say, a factor of 5, unless we make very unreasonable or counterintuitive assumptions?

To analyze distributional concerns in CBA outside the special assumption used by Hylland and Zeckhauser (1979) and followers is the main purpose of this paper. A more specific aim is to investigate whether distributional concerns are warranted in CBA, and if so, to what extent, for the natural benchmark case when different publicly provided goods affect tax revenues, through possible induced labor supply adjustments, equally much per dollar spent on each good. As demonstrated in Section 2, it turns out that distributional weights are warranted then, and that the weights should be inversely related to the social marginal utility of income, implying that they will vary largely with income for empirically realistic values of individual risk aversion. A sufficient condition for this is that utility is separable in the publicly provided goods, instead of in leisure. These results hold for a wide range of tax structures, including poll taxes and linear, piece-wise linear, and non-linear income taxes, and regardless of whether the taxes are optimal or not. The different separability assumptions are compared in Section 3, and it is argued that there is little substantial basis to argue that one separability structure is more plausible than the other as a general benchmark case. Moreover, it is concluded that using distributional weights is always optimal, irrespective of specific preference structures, unless there are differences in induced tax revenue effects between the goods that work in the direction of off-setting the distributional welfare effects.

In Section 4, we discuss cases that may seem to imply contradictions with earlier theoretical results. First, we analyze strong (additive) separability in the publicly provided goods, which is a case studied by Broadway and Keen (1993). They showed that the condition for when the good should be over-provided relative to the Samuelson rule is independent of the distributional characteristics of the good. Second, we analyze a particular functional form discussed by Kaplow (1996), where both types of weak separability hold simultaneously. Since strong separability implies weak separability, it seems that for both of these cases we should simultaneously apply, and not apply, distributional

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7 Diamond (1975) and Atkinson and Stiglitz (1980, 496–97) have shown that distributional weights should be applied, given this type of separability, in the special case of optimal linear income taxation.
weights, which seems to imply a contradiction. However, as will be shown, this is an illusion since for both of these special cases the marginal WTP for all public goods will vary with income in an identical manner, so that applying weights will make no difference in terms of allocations. Consequently, the allocation would be exactly the same with and without weights in these special cases, implying both that distributional weights are redundant and that they are harmless in terms of efficiency losses. Section 5 summarizes and discusses the results in general within a broader policy context.

II. CHOOSING BETWEEN PUBLIC PROJECTS

As the theoretical point of departure, we will use a standard public economics setup in the tradition of Mirrlees (1971), where the government’s objective is to maximize a general Bergson-Samuelson SWF:

\[ w(U^1, U^2, ..., U^n), \]  

subject to a public budget constraint. \( w \) is assumed to be increasing and weakly quasi-concave in its arguments, that is, the individual utilities, where \( U^i \) is utility for individual \( i \). The government chooses the amount to provide of \( m \) different goods (or more generally projects, they need not be pure public goods), such as environmental goods, with different distributional characteristics; some may be preferred mainly by poor people and some by the rich. To simplify presentation, it is assumed that the production technology for these goods is linear, so that the production prices of these goods are fixed. Individuals get utility from private consumption \( x \) and leisure \( l \) in addition to the publicly provided goods:

\[ U^i = u^i(x^i, l^i, G_1, ..., G_m). \]  

Individual \( i \)'s gross income is given by \( a^iL^i \), where \( a^i \) is the exogenously given “ability” or productivity, which in equilibrium can be seen as the net wage level, and \( L^i \) is the chosen amount of working time to supply. Naturally, the total available amount of time, \( \omega \), is limited so that

\[ l^i = \omega - L^i. \]  

The individual’s consumption is given by the gross income minus the income tax \( T(a^iL^i) \), where the tax function \( T \) is piecewise continuously differentiable. Thus we have

\[ x^i = a^iL^i - T(a^iL^i). \]  

At this stage we do not need to specify the tax function \( T \) further—it may be optimal or non-optimal, linear, piecewise linear, or non-linear. However, it is essential that the tax can be related only to gross income, and it is assumed to be impossible for the government to directly observe the individual ability or wage level as is standard in the optimal taxation literature. Consequently, we have a second-best problem where the individual choice of working time (and hence leisure) is generally not socially optimal. Substituting [3] and [4] into [2] we get

\[ U^i = u^i(a^iL^i - T(a^iL^i), \omega - L^i, G_1, ..., G_m), \]  

implying the following individual optimum condition with respect to the choice of \( L \):

\[ \frac{\partial u^i}{\partial x}(1 - t^i) = \frac{\partial u^i}{\partial l^i}, \]  

where \( t^i = \frac{\partial T}{\partial a^i L^i} \) is \( i \)'s marginal tax rate.

The government’s objective is to maximize social welfare, that is, eq. [1], subject to an exogenous public budget balance requirement \( R_0 \) (which can take any value, including zero), implying the following Lagrangean:

\[ w(U^1, ..., U^n) + \lambda \left( \sum_i T(a^iL^i) - \sum_j p_j G_j - R_0 \right), \]
where \( p_j \) is the production price of good \( j \). The social first order optimum condition for the provision of an arbitrary good 1 is then given by

\[
\sum_i \frac{\partial w}{\partial U_i} \frac{\partial u_i}{\partial G_i} + \lambda \left( \sum_i t'a_i \frac{\partial L_i}{\partial G_i} - p_i \right) = 0, \tag{8}
\]

which can be re-written as

\[
\sum_i \alpha_i^tMRS_{G,t} = \lambda (p_t - \partial R/\partial G_1), \tag{9}
\]

where \( \alpha_i^t = \frac{\partial w}{\partial U_i} \frac{\partial u_i}{\partial x_i} \) is the social marginal utility of income, that is, how much an additional dollar of consumption to individual \( i \) contributes to social welfare; \( MRS_{G,t} = \frac{\partial u_i}{\partial G_i} / \frac{\partial x_i}{\partial G_i} \) is individual \( i \)'s marginal rate of substitution between the publicly provided good 1 and \( x_i \) (i.e. the marginal willingness to pay for good 1 in terms of \( x_i \)); and where we denote \( \frac{\partial R}{\partial G_1} = \sum_i t'a_i \frac{\partial L_i}{\partial G_1} \), the induced revenue effect caused by good 1, that is, how much the tax revenues change due to the induced changes in the supplied labor of a marginal increase in the provision of good 1. Combining the first order conditions for goods 1 and 2 then implies

\[
\sum_i \alpha_i^t MRS_{G,t} = \frac{\sum_i \alpha_i^t MRS_{G,t}}{p_1 - \partial R/\partial G_1} = \frac{\sum_i \alpha_i^t MRS_{G,t}}{p_2 - \partial R/\partial G_2}. \tag{10}
\]

This result can be compared with the basic first-best Samuelson (1954) rule implying that

\[
\frac{\sum_i MRS_{G,t}}{p_1} = \frac{\sum_i MRS_{G,t}}{p_2}. \tag{11}
\]

Thus, we have two deviations from the basic Samuelson rule. The rationale behind the \( \alpha \)-factors is straightforward: Since one additional dollar to a poor individual typically implies more social welfare, \textit{ceteris paribus}, than one additional dollar to a rich individual, we need the \( \alpha \)-factors as conversion factors, or distributional weights, to correspondingly translate the monetary marginal WTP measures into social welfare units.

The rationale behind the additional term in the denominators is almost as straightforward: Equation [6] implies that each individual (who is working a non-zero amount) on the margin will be indifferent between working one more hour or not. However, the income tax drives a wedge between the private optimum and the social optimum. If individual \( i \) works one more hour, the society obtains an additional tax revenue equal to \( t'a_i \), which is of course beneficial for the society as a whole. Hence, the society prefers each individual, on the margin, to work more. Consequently, a marginal increase in the public provision of good 1 implies, for a given tax schedule, that the tax revenues change with \( \frac{\partial R}{\partial G_1} = \sum_i t'a_i \frac{\partial L_i}{\partial G_1} \) for the society as a whole, that is, due to the change in all individuals' supplied amount of labor. Again, the utility change of these marginal changes in working hours for each individual is zero, implying that there is no corresponding welfare costs to these tax revenues. Thus, the optimality condition [10] simply implies that the amount of social welfare obtained per unit of dollar-reduction in the social budget should be the same for each good provided. The cost in terms of dollar-reduction in the social budget for good 1, in turn, is equal to \( p_1 - \partial R/\partial G_1 \), that is, the production cost of good 1 minus the changes in

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\(^9\) Following the terminology of Diamond and Mirrlees (1971), and hence not the one of Diamond (1975). For a utilitarian SWF we have of course that \( \alpha_i^t = \partial u_i/\partial x_i \), i.e., the individual marginal utility of income.

\(^{10}\) Note that this conclusion does not depend on whether an increased tax level actually decreases the supplied amount of labor or not, i.e., whether the substitution effect or the income effect dominates. Still, the conclusion depends of course on the (standard) assumptions made. For example, if people also care about relative consumption, i.e., their own consumption relative to those of others, which appears reasonable given existing empirical evidence as well as common-sense reasoning, it is not obvious that it would be socially beneficial if people worked more on the margin; compare with Brekke and Howarth (2002), Johansson-Stenman, Carlsson, and Daruvala (2002), and Alipizar, Carlsson, and Johansson-Stenman (2005).
tax revenues that the marginal provision induces.

Although distributional weights do generally seem to play a role in [10], through the \( \alpha \)-factors in the nominators, it is not possible to generally say from this equation whether a good preferred by the poor should be over-provided relative to the other good, compared to the basic efficiency rule. The reason is that although the WTPs by the rich are multiplied by smaller \( \alpha \)-factors, it is possible that this effect interacts with the tax-revenue effect, which we may alternatively call the efficiency effect, in terms of changes in labor supply. For example, it is possible that the good preferred by the rich induces people to work more on the margin, while the other good has the opposite effect. Whether these effects interact or not depends crucially on people’s preference structure, and we will see that different commonly made preference assumptions imply very large differences in policy conclusions regarding the appropriate use of distributional weights.

Let us now consider the natural benchmark case where the tax-revenue effects are the same for per unit of dollars spent on goods 1 and 2, that is,

\[
\frac{1}{p_1} \sum_i t^i a^i \frac{\partial L^i}{\partial G_1} = \frac{1}{p_2} \sum_i t^i a^i \frac{\partial L^i}{\partial G_2}.
\]

[12]

In this case we have, from substituting [12] into [10], that

\[
\frac{\sum_i \alpha^i MRS^i_{G_1}}{p_1} = \frac{\sum_i \alpha^i MRS^i_{G_2}}{p_2}.
\]

[13]

This result implies that for any size of the public budget (optimal or not), and for any taxation system (optimal or not), we should choose the combination of \( G_1 \) and \( G_2 \) per dollar spent that contributes most to social welfare \textit{ceteris paribus}, since there are no indirect tax-revenue effects through changed labor supply to correct for. In doing so, we need distributional weights, the \( \alpha \)-factors, to translate the monetary units of the marginal willingness to pay measures into social welfare units. That is, we need to take the fact into account that a certain WTP of a poor person reflects a higher welfare effect than the same WTP of a rich person.

\section*{Separability in Publicly Provided Goods}

A sufficient, but not necessary, condition for equal tax-revenue effects is clearly that these effects are zero for both goods, that is, both the left-hand-side and the right-hand-side of equation [12] are equal to zero. A sufficient, but not necessary, condition for this, in turn, is that these effects are zero for each individual, that is, that there are no labor supply effects of the public goods \textit{per se}, so that \( \partial L^i / \partial G_1 = \partial L^i / \partial G_2 = 0 \), for all individuals. A sufficient, and necessary, \textsuperscript{12} condition for this, in turn, is the frequently used structure where utility can be written as weakly separable in the publicly provided goods:

\[
U^t = u^t(x^i, l^i, G_1, ..., G_m).
\]

[14]

This formulation implies that the marginal rate of substitution between leisure and private consumption is unaffected by the provision of the publicly provided goods, and the maximization of [14] for the individual simply implies the maximization of the sub-utility function \( f(x^i, l^i) \), which is clearly independent of the publicly provided goods. Hence, the (uncompensated) labor supplied is also independent of these goods, so that \( \partial L^i / \partial G = 0 \) for all individuals \( i \) and public goods \( j \), which implies that equation [13] holds. Consequently, rather than comparing the aggregate marginal willingness to pay for the public goods, we should compare the weighted sums, where the weights are given by the social marginal utility of income. Again, since this holds generally (given the assumed separability), it also holds for optimal non-linear income taxes. Alternatively, we can rewrite [13] to

\textsuperscript{11} In the case of optimal taxation, our results hold also for a tax reform, by duality, so that it is always optimal to use these distributional weights also if one can change the income tax structure. This is not necessarily true if the initial income tax is not optimal, however.

\textsuperscript{12} See Goldman and Uzawa (1964).
separate out the distributional effects. Using the definition of the expected value of a product, that \( E(\alpha \ MRS) = E(\alpha)E(MRS) + \text{cov}(\alpha, MRS) \), we have that

\[
\sum_i \alpha_i \ MRS_{\alpha, i} = \sum_i MRS_{\alpha, i} \left[ 1 + \text{cov}(\frac{\alpha_i}{\alpha}, \frac{MRS_{\alpha, i}}{MRS}) \right] \bar{\alpha} = \sum_i MRS_{\alpha, i} (1 + \delta_i) \bar{\alpha},
\]

where an overbar denotes mean value and where

\[
\delta_i = \text{cov}(\frac{\alpha_i}{\alpha}, \frac{MRS_{\alpha, i}}{MRS}) \quad \text{[16]}
\]


\[
\sum_i MRS_{\alpha, i} = \frac{p_1}{p_2} \frac{1 + \delta_1}{1 + \delta_2} \quad \text{[17]}
\]

Hence, the ratio between the sums of the \( MRS \) should not equal the \( MRT \) ratio, that is, the production cost ratio, but rather the \( MRT \) ratio times an expression which compares the distributional characteristics, or the normalized covariances between the marginal willingness to pay for the public goods and the social marginal utility of income. Goods that are relatively more preferred by low-income people should then be over-provided compared to other goods, and vice versa.

It should be noted that although we allow for heterogeneity in preferences, the results hold also when the preferences are identical in the population, as shown in Appendix 1. Hence, the case for distributional weights is not driven by any implicit assumption that poor people have different preferences than the rich.

A Transport Example

Consider two small\(^{13} \) and equally costly public projects to be compared: one improving the local road infrastructure, and one improving the public transport system. Both projects are assumed to influence tax revenues, through labor supply effects, about equally much. The latter project is preferred primarily by low-income people, since most high-income people will continue not to use public transport irrespective of improvements. Surveys have been undertaken to elicit people's marginal WTPs for small improvements with equal cost in these two areas, and aggregate WTP is found to be 50% higher for the improved road infrastructure. As is standard in survey analysis, people's incomes (net of taxes) were asked for, and the WTP income elasticities for these projects, defined as

\[
\varepsilon = \frac{\partial MRS_{\alpha,i}}{\partial x} \frac{x}{MRS_{\alpha,i}},
\]

were estimated. The income elasticity was found to equal 1 for the road infrastructure investment, and 0 for the public transport investment. The after-tax income in the economy is assumed to be Gamma-distributed and given by

\[
f(x) = xe^{-x},
\]

where \( x \geq 0 \), implying a Gini-coefficient of about 0.37, which is of the same order of magnitude as for many western European countries. The social welfare function is assumed to be utilitarian, and the representative utility function can be characterized by constant relative risk aversion, implying that

\[
a(x) = x^{-\sigma},
\]

where \( \sigma \) is the social inequality aversion, measured as individual relative risk aversion in income. Let us conservatively assume \( \sigma \) to equal 1.

Which project should be preferred? Given constant income elasticities, we can substitute [18–20] into a continuous version of equation [15] and write the distributional characteristic for the publicly provided good as

\footnote{\(^{13} \text{Meaning that the income effect of the good provision is negligible so that the continuous case is a good approximation.} \)}
\[ \delta = \frac{\int_0^\infty \alpha MRS_G f(x) \, dx}{\int_0^\infty MRS_G f(x) \, dx} - 1 \]
\[ = \frac{\int_0^\infty x^{1-\sigma} e^{-x} \, dx}{\int_0^\infty x^{\sigma} e^{-x} \, dx} - 1. \tag{21} \]

Imposing the values for \( \alpha \) and \( \sigma \) into (21), we get that the distributional characteristics are equal to \(-0.5\) for road infrastructure, and \(0\) for public transport. Imposing these values into (17), where good \( j \) is the local road improvement, we see that the right-hand-side ratio is equal to \(2\), which is larger than the aggregate marginal WTP ratio of \(1.5\). Thus, the road improvement is in this case less socially profitable than the public transport improvement, despite the fact that it would have been more profitable had we only considered efficiency aspects. Expressed alternatively, the size of the pie in terms of social welfare for a given degree of inequality will in this case be larger with distributional weights than without them, even when we have the possibility to use optimal non-linear income taxes to deal with equity.

**Separability in Leisure**

Consider now instead the utility function discussed by Christiansen (1981) and others where utility is separable in leisure as follows:

\[ U^j = u(f(x^j, G_1, \ldots, G_n), l^j), \tag{22a} \]

or the slightly more general version discussed by Broadway and Keen (1993):

\[ U^j = u^i(f(x^j, G_1, \ldots, G_n), l^j). \tag{22b} \]

Such preferences together with optimal non-linear income taxes, as thoroughly demonstrated in different ways by the above authors, imply that the basic Samuelson rule for the provision of public goods still holds, that is, it is optimal not to use any distributional weights. In order to show that result here, we will adopt a somewhat different strategy than in the previous case.

Consider a combined provision of a good \( j \) and its financing, taking possible revenue effects through labor-supply adjustments into account, and denote this combined provision and financing \( Q_j \). That is, a marginal change in \( Q_j \), implying an equally large change in \( G_j \), keeps the public budget unchanged. Without specifying how the goods are financed, we can write the marginal social welfare effect of a marginal change in \( Q_j \) as

\[ \frac{\partial w}{\partial Q_j} = \sum_i \frac{\partial w}{\partial U^i} \left( \frac{\partial u^i}{\partial x^i} \frac{\partial x^i}{\partial Q_j} + \frac{\partial u^i}{\partial l^i} \frac{\partial l^i}{\partial Q_j} + \frac{\partial u^i}{\partial G_j} \right) \]
\[ = \sum_i \alpha^i \left( \frac{\partial x^i}{\partial Q_j} + MRS_{x^i} \frac{\partial l^i}{\partial Q_j} + MRS_{l^i} \right), \tag{23} \]

where \( MRS_{x^i} \) is individual \( i \)'s marginal value of leisure time in terms of \( x \). At the social optimum, given any particular way to finance the good, this expression must clearly equal zero. An optimal non-linear income tax system implies that any combination of marginal tax parameter changes to raise an additional dollar of revenue is equally good (or bad) in terms of social welfare (by the envelope theorem).\(^14\) Let us now consider a particular way to finance a marginal increase of the good, namely a change in the tax function so that each individual \( i \), through increased income taxes, pays exactly his/her marginal WTP for the good \( j \), so that we have

\[ - \frac{\partial x^i}{\partial Q_j} = MRS_{x^i}. \tag{24} \]

Let us now analyze labor adjustments of this combined provision and financing. Both [22a] and [22b] imply that we can write an inverse function as

\[ l^j = \psi(U^j, f^j) = \psi(U^j, f(x^j, G_1, \ldots, G_n)), \tag{25} \]

where \( \psi \) is monotonically increasing in \( U^j \) and decreasing in \( f^j \). This formulation implies that any changes in the arguments of \( f \) that keep \( f^j \) unchanged will imply that \( l^j \),

\(^{14}\) Otherwise the first-order optimality conditions would not be fulfilled, and the tax system should be adjusted so that they are fulfilled.
and hence labor, will be unchanged too. Since each individual pays his/her marginal WTP for the increased provision, this means that \( f' \) is unchanged. Consequently, each individual will choose exactly the same amount of leisure (and labor) as before this reform, implying that

\[
\frac{\partial l}{\partial Q_j} = 0, \tag{26}
\]

for all individuals \( i \) and financed provisions \( j \). Since the budget is balanced on the margin, this way of financing then also implies that the good will be provided until the sum of the individuals’ marginal WTPs for the good equals the per-unit price of it, that is, it implies that the Samuelson rule is applied. In order to see that this provision is socially optimal, substitute [24] and [26] into [23]. This implies that the expression in parentheses in [23] equals zero, and hence that \( \frac{\partial w}{\partial Q_j} = 0 \), so that the social optimum condition is fulfilled. Thus, we have shown that the basic Samuelson rule, without any distributional concern, is indeed optimal to use in this particular case.

III. COMPARING THE ASSUMPTIONS AND THEIR PLAUSIBILITY

Intuitively, why are the policy conclusions so very different for the two commonly used types of separability assumptions discussed? In the case of separability in the publicly provided goods, as in [14], an increase in these goods per se does not affect labor choices. Hence, the distributonal welfare effects should in this case be taken care of in full, since there are no incentive effects, or “distortionary cost,” from the provision of these goods per se.

In the case of separability in leisure, as in [22a and 22b], on the other hand, a combined public good and tax increase that is exactly equal to the marginal WTP for all leaves the labor choice unaffected. Thus, a marginal provision increase together with this particular way of financing it have neither any distributional effects nor any efficiency (or tax-revenue) effects on the margin. Intuitively, if we can ignore both induced efficiency effects and distributional effects, then the first best rule can be applied, and hence the efficiency-based Samuelson (1954) rule is optimal. This also means that if one good has large distributional benefits compared to another good, these benefits will always be exactly offset by larger efficiency costs in terms of induced labor supply effects.

Given the large differences in policy conclusions it is of course important to judge which case is more reasonable, and which assumption should be seen as the most natural benchmark case. There is unfortunately little empirical evidence that can be used to compare the realism of these assumptions with each other, and none of the papers cited above refers to any empirical evidence supporting their assumptions. Kaplow (1996, 2004), for example, refers to separability in leisure as part of “standard simplifying assumptions.” On the other hand, Starret (1988, 173) argues that “a general project has no obvious net complementarity,” supporting separability in the publicly provided goods.

To shed some light on the realism of these assumptions, we will briefly discuss three examples. Consider first a transport-related example that reduces commuting time to work, but has no other effects. It seems reasonable that part of the corresponding time gains may be used to increase working hours, and part to increase leisure. This is consistent with separability in leisure, which gives no prediction regarding sign,\textsuperscript{15} but is not consistent with separability in the publicly provided goods, which predicts no change in working hours. However, separability in leisure implies that the value put on the time reduction is independent of the amount of leisure, which seems unreasonable; one would instead believe that the value of time would decrease with the amount of leisure ceteris paribus. This prediction is instead

\textsuperscript{15} Remember that the overall effects of the tax increase and marginal good provision leave the amount of labor unchanged. Since the tax increase on its own can either increase or decrease labor, depending on the relative size of the income and substitution effects, the sign regarding the good provision can also be either positive or negative.
consistent with separability in $G$. Hence, in this example, none of the assumptions seems to offer a good approximation of reality.

Next, consider police and defense investments. Presumably, such investments have quite minor effects on the chosen provided amount of labor, which is consistent with both separability assumptions. Separability in leisure also predicts individuals' valuation of an increased public investment to be independent of leisure, which seems reasonably plausible as well.

Finally, consider investments in so-called leisure goods such as theaters, and other cultural or recreational investments that make leisure relatively more attractive. Such investments per se tend to increase the chosen amount of leisure, which is consistent with separability in leisure but not with separability in $G$. However, again separability in leisure implies that the monetary value put on increased $G$ is independent of the amount of leisure. But in reality one would, of course, expect the value put on an increased provision of a leisure good to increase with the amount of leisure, everything else held constant. Indeed, that is why they are called leisure goods—after all, what is the benefit of increased provision of leisure goods if you have no leisure? Hence, in this example too, none of the assumptions seems to offer a good approximation of reality. Thus, neither of these two frequently made separability assumptions seems to give a representative picture of most public goods, and it appears likely that different assumptions will be more or less suitable for different goods. Which case should serve as the most natural "benchmark" appears to be an open question.

However, it should again be pointed out that separability in the publicly provided goods is a sufficient, but far from necessary, condition for equal tax-revenue effects between the provided goods. The assumption that two provided goods have equal tax-revenue effects, per dollar spent on them, is therefore much less restrictive than assuming utility functions that are weakly separable in both publicly provided goods.

Moreover, not even equal revenue effects are necessary for the conclusion that taking distributional considerations in the goods provisions is welfare improving. Indeed, recall that [10] implies that distributional adjustments should be made unless the induced efficiency welfare effects, through the tax-revenue effects, offset the distributional welfare effects. However, it is not even clear that these differences must counteract the distributional welfare effects, even when we do not have equal revenue effects. Indeed, it is easy to think of examples where the distributional welfare effects of using weights are not at all offset by efficiency-based welfare effects (as is the case given the assumptions used by Christiansen and others) but rather amplify the welfare effects by the distributional differences.

Consider a governmental choice between improvements of the local golf course and improvements of the public water-supply system. Naturally, the rich would prefer the golf-course improvement, since they have all invested in private high-quality water supply systems, while the poor would prefer the water-supply improvement, for example, since they cannot afford to play golf anyway. Here we have that the efficiency effects of the project preferred by the rich are expected to be negative, since the golf course improvement makes leisure relatively more attractive.¹⁶ The water supply improvement, on the other hand, can be expected to generate an increased amount of supplied labor due to reduced water-related morbidity. Thus, in this case the good preferred by the poor should clearly be over-provided relative to the other good, compared to the implications of the pure efficiency WTP rule. Moreover, it should also be over-provided relative to [13] or [17], which do include distributional weights, since the induced efficiency effects enhance the distributional welfare effects.

This clearly highlights that the assumptions underlying the conclusion that one should generally focus on efficiency-only,

¹⁶ Moreover, the efficiency effects are even worse due to the fact that it is the rich persons who work less, since they pay higher taxes per hour.
as argued by Kaplow (1996, 2004) and others, are indeed very strong, as noted also by Sandmo (1998) and Slemrod and Yitzhaki (2001), and that they sometimes imply extremely bad approximations of reality. Thus, even though it appears difficult to judge how reasonable, or unreasonable, different preference structures are, it is from a policy perspective sufficient to be able to judge the relative tax-revenue effects. And as long as the good preferred from a distributional point of view is not substantially worse from an efficiency point of view through induced labor supply effects, it is optimal to use distributional weights.

IV. SPECIAL CASES

Additive Separability in Public Goods

Boadway and Keen (1993) derived, using the self-selection approach, a result for the case where the (common for all) utility function is additively, or strongly, separable (and hence also weakly separable) as follows

$$U^t = A(x^1, l^t) + B(G_1, \ldots, G_m).$$  \[27\]

They showed that, given optimal non-linear income taxes, a public good should be over-provided relative to the Samuelson rule if \(\frac{\partial^2 A}{\partial x \partial l} < 0\), that is, if private consumption and leisure are Edgeworth complements, and vice versa. This condition is clearly independent of the distributional characteristics of the publicly provided goods. Thus, there is no explicit equity consideration here, despite the weakly separable public goods, which seems to contradict the results in [17]. However, as will be shown, this is an illusion. Individual \(i\)'s marginal WTP for an increase in the provision of good \(j\) is given by

$$\text{MRS}_{x^i, j} = \frac{A}{\partial G_j/\partial x^i}. \quad [28]$$

Since the utility function is the same for all, we can then use [28] to calculate the corresponding income elasticity in the population as

$$\varepsilon_j = \frac{\partial MRS_{x^i}}{\partial x} \frac{x}{MRS_{x^i}} = \frac{\partial A}{\partial x^i} \frac{\partial A}{\partial x}. \quad [29]$$

which is clearly independent of all publicly provided goods. Hence, the marginal WTP varies with income in exactly the same way for all public goods. Alternatively, we can combine [28] and [15] and write the distributional characteristic of good \(j\) as\(^{17}\)

$$\delta_j = \frac{\sum \alpha_i \frac{\partial B}{\partial G_j} / \partial x^i}{\sum \alpha_i \frac{\partial A}{\partial x^i}} - 1 = \frac{\sum \alpha_i \frac{\partial A}{\partial x^i}}{\sum \alpha_i} - 1, \quad [30]$$

which is clearly independent of all publicly provided goods, implying that the distributional characteristics will be the same for each of these goods, and the right hand side of [17] will then simply be equal to the cost ratio. Thus, given preferences as in [24], comparing two (small) public projects in terms of the cost-benefit ratio will give the same result with and without distributional weights.

Separability in Both Leisure and Publicly Provided Goods

Consider now the special case discussed by Kaplow (1996) where people’s utility functions are given by

$$U^t = f(x^1) + g(G_1, \ldots, G_m) + h(l^t). \quad [31]$$

This utility function is separable in both the publicly provided goods and in leisure. As we have seen, according to the former we should apply distributional weights, but according to the latter we should not.

\(^{17}\) Assuming a utilitarian SWF we get that \(\delta = \frac{1}{\mu 1/\mu_1} - 1\), where \(\mu\) is the individual marginal utility of income. Hence, the larger the (after-tax) inequality, the larger the distributional characteristic (in absolute value), reflecting the fact that the public good will then have a more equalizing effect.
which seems again to indicate a contradiction. However, since [31] is a special case of [27], the same arguments are applicable here. Hence, [17] and the Samuelson rule will hold simultaneously. In Appendix 2, it is shown that a sufficient condition for this result is that utility is weakly separable in both leisure and the public goods, simultaneously, so that the utility function can be written as

\[ U' = u(f(x', l'), G_1, \ldots, G_m) = v(h(x', G_1, \ldots, G_m), l'). \]

[32]

Consequently, the allocation would be exactly the same with and without weights in these special cases, implying that distributional weights are redundant, but also that they are harmless in terms of efficiency losses.

V. DISCUSSION AND CONCLUSIONS

This paper has questioned the proposition that distributional weights in CBA typically imply large inefficiency losses, compared to other ways of dealing with equity such as income taxes. Instead it is shown that it is optimal to use such weights in the choice between different publicly provided goods (or projects, more generally), when these goods are expected to affect tax revenues, through changes in labor supply, about equally much per dollar spent on them. For this conclusion, we have not assumed anything further regarding people’s preference structures or the characteristics of the implemented tax structure. Still, it has been shown that preferences that are weakly separable in the publicly provided goods are sufficient conditions. Moreover, it has been shown that distributional concerns should be taken unless differences in distributional welfare effects are offset by corresponding differences in tax-revenue effects. This would occur given a set of assumptions used by several authors, but it is demonstrated that these assumptions often constitute very poor approximations of reality.

Of course, if one does not care about equity at all at the social level, then introducing ad hoc distributional weights will typically imply large inefficiency losses. Harberger (1978, 1984) provides several examples of the negative consequences of applying distributional weights in cost-benefit analysis. One of the more amusing ones deals with the possibility of sending ice-cream on camel-back across the desert, from a richer oasis to a poorer one (Harberger 1978, repeated in Harberger 1984). In an extreme case, when the social inequality-aversion used is large, he asserts (Harberger 1978, S113) that “up to 63/64 of the ice-cream could melt away without causing the project to fail the test.” He concludes (Harberger 1984, 458): “Even ways which by the traditional standards would be scandalously inefficient would have to be explored.”

However, comparisons with a first-best world is of little help, given that the society cares about equity as reflected by an SWF which is reasonably quasi-concave in income. Decreasing inequality by changes in income taxation will then have large inefficiency losses as well, as pointed out by Dahlby (1998) and Sandmo (1998). To illustrate this, consider again the initial example with a utilitarian SWF, where each individual has an identical utility function characterized by constant relative risk aversion, and where the income of a rich person is 100 times that of a poor. Everything else being equal, a redistribution of one dollar from the rich to the poor through changed income taxes would, in the absence of any inefficiency losses, give the poor individual a utility increase which is 100\(\gamma\) times the utility loss of the rich individual, where \(\gamma\) is again the (constant and common for all) parameter of relative risk aversion. Such redistributions without any efficiency costs are in most cases unrealistic, however. Assume therefore that only the fraction \((1 - \gamma)\) reaches the poor recipient, where \(\gamma\) is the inefficiency loss or the marginal excess burden. Then the social welfare change of the redistribution is positive if (and only if) \((1 - \gamma)100^\sigma > 1\). Consequently, this redistribution would be social-welfare improving if the marginal excess burden of this tax change were less than 99% for \(\sigma = 1\), and 99.99% for \(\sigma = 2\). Indeed, from a
normative perspective one may even argue that these inefficiency losses should be large, given what can be considered to be a reasonable SWF—otherwise the redistributive taxes are simply too low. From a dual formulation of the welfare maximization, where the size of the economy is maximized given a certain inequality level, it follows that these losses are sometimes lower when only making tax changes, and sometimes they are lower when also taking distributional considerations in CBA.

In this study, as in most other studies, aspects such as simplicity of analysis, administrative costs, and implementation issues are ignored. Presumably, such considerations are often important reasons for not using distributional weights in practice, but sometimes there are also practical arguments in favor of using such weights. The income-tax system works for example very poorly in many developing countries, and equity concerns directed through public projects may then have quite a high distributional accuracy compared to the available alternatives. There are also instances where pure efficiency arguments appear particularly hard to digest. According to the empirical findings of Miller (2000), the WTP-based value of a statistical life is roughly proportional to income. A pure efficiency-based CBA then implies that saving one person with income 10x is preferable to saving nine persons with income x (each). In practice, it is hard to imagine that any publicly funded institution uses such priorities and it would be very hard for them to defend them; see for example Usher (2001) and Sunstein (2004).

Finally, the analysis in this paper takes the ethics underlying the (quite flexible) Bergson-Samuelson SWF as a point of departure, and the distributional weights are of course derived conditional on the underlying assumptions. Still, even though many alternative ethical assumptions do exist, it is hard to imagine a consistent and defendable ethical theory that implies per se that efficiency-only should guide public policy. Moreover, even though the specific way in which distributional concerns should be made may vary with the alternative ethical assumptions, one can generally not escape the fact that efficiency and distributional concerns must be analyzed simultaneously.

In conclusion, it appears difficult to defend both the proposition that distributional concerns should always be used in CBA, and that they should never be used. However, the commonly expressed perception that using distributional weights generally imply large efficiency losses appears to be incorrect, or strongly misleading at best.

APPENDIX 1

SEPARABILITY IN THE PUBLICLY PROVIDED GOODS AND UNIFORM PREFERENCES

When the preferences are identical in the population and separable in the publicly provided goods, they can be written as follows for an individual i:

\[ U^i = u(x^i, l^i, G_1, \ldots, G_n). \]  \hspace{1cm} [A1]

There are many utility functions of this kind that imply different income elasticities for the publicly provided goods, and hence different distributional characteristics. Consider for example the following utility function:

\[ U^i = g(G_1(G_2 + x^i l^i)), \]  \hspace{1cm} [A2]

where g is any increasing monotonic transformation (to ensure suitable concavity, etc.). From individual utility maximization with respect to how much to work, we have from eq. [6] that

\[ l^i = \frac{x^i}{a'(1 - t^i)}, \]  \hspace{1cm} [A3]


\[ U^i = g\left(G_1\left(G_2 + \frac{x^i l^i}{a'(1 - t^i)}\right)\right). \]  \hspace{1cm} [A4]

The marginal WTP ratio between the publicly provided goods 1 and 2 is then given by

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18 For a critique of ethical justifications of the Hicks-Kaldor compensation criteria, see e.g., Sen (1987).
\[
\frac{\text{MRS}_{x_i, t}}{\text{MRS}_{G_i, t}} = \frac{\frac{\partial u}{\partial G_i} (G_2 \cdot \frac{x_i}{a'} (1 - t))}{G_1}.
\]

This is clearly increasing in after-tax income \(x_i\), provided that utility is increasing in after-tax income which is typically assumed. Thus, a richer individual will always value a small increase of \(G_i\), in terms of \(G_1\), higher than a poorer individual will, implying different distributional characteristics. This demonstrates that the ease for distributional weights is not driven by any implicit assumption that poor people have different preferences than the rich.

APPENDIX 2

WEAK SEPARABILITY IN BOTH LEISURE AND PUBLICLY PROVIDED GOODS

A utility function that is weakly separable in both leisure and the publicly provided goods, and which is the same for all, can be written as

\[
u(f(x', l'), G_1, ..., G_m) = v(h(x', G_1, ..., G_m), l').
\]  \[A7\]

Using the latter function, the ratio between the marginal WTP for goods 2 and 1 can then be written

\[
r = \frac{\text{MRS}_{x_i, G_i}}{\text{MRS}_{x_i, l}} = \frac{\frac{\partial h}{\partial G_i}}{\frac{\partial h}{\partial l}}.
\]  \[A8\]

which is a function of \(x_i\), and \(G_1, ..., G_m\), and is clearly independent of \(l\). But, using the former function, this ratio can also be written

\[
r = \frac{\frac{\partial u}{\partial G_i}}{\frac{\partial u}{\partial G_1}},
\]  \[A9\]

which is a function of \(f(x', l')\) and \(G_1, ..., G_m\). By combining these implications of [A8] and [A9], we can write

\[r' = s(x', G_1, ..., G_m) = v(f(x', l'), G_1, ..., G_m).\]  \[A10\]

Differentiating [A10] with respect to leisure implies that

\[
\frac{dr'}{dl'} = \frac{\partial s}{\partial l'} = \frac{\partial v}{\partial f} \frac{\partial f}{\partial l'}.
\]  \[A11\]

Since \(s\) is independent of leisure we have that \(\frac{\partial s}{\partial l'} = 0\), which then clearly implies that

\[
\frac{\partial v}{\partial f} = 0.
\]  \[A12\]

But since people’s marginal utility of leisure is in general non-zero, so that \(\frac{\partial f}{\partial l'} \neq 0\), we must have that

\[
\frac{\partial r}{\partial f} = 0.
\]  \[A13\]

Combining [A10] and [A13] implies that \(r'\) is independent of \(x'\) as well. Thus, \(r'\) can be written as a function solely of \(G_1, ..., G_m\) for all individuals \(i\). This means that all individuals, having the same utility function, will have the same \(r\). The distributional characteristics are then the same for goods 1 and 2, since

\[
\delta_2 = \text{cov} \left( \frac{\alpha}{\alpha} \frac{\text{MRS}_{x_i, G_i}}{\text{MRS}_{x_i, l}} \right) = \text{cov} \left( \frac{\alpha}{\alpha} \frac{r\text{MRS}_{G_1}}{\text{MRS}_{G_1, l}} \right)
\]  \[A14\]

which should be shown.

References


