This article concerns optimal income taxation under asymmetric information in a two-type OLG model when individuals’ relative consumption matters. Positional concerns affect the policy choices via two channels: (i) the average degree of positionality and (ii) positionality differences between the low-ability type and the mimicker. Under plausible empirical estimates, the marginal labor income tax rates become substantially larger, and the absolute value of the marginal capital income tax rate of the low-ability type becomes substantially smaller, than in the conventional model. In addition to measures of reference consumption based on average consumption, we also address within-generation and upward comparisons.

1. INTRODUCTION

Since the late 1970s, a literature dealing with public policy in economies where consumers have positional preferences, that is, relative consumption concerns, has gradually developed. The importance of this literature has become more apparent over time, as corresponding empirical literature has grown. There is by now convincing empirical support for the idea that relative consumption comparisons are important from at least three independent economic subliteratures: happiness research, questionnaire-based experiments, and more recently from brain science. There are also recent evolutionary models consistent with relative consumption concerns.

Earlier studies on optimal taxation in economies where people make relative consumption comparisons often assume that the government uses linear tax instruments. Furthermore, almost all of them are based on static models, and have in common that they neglect capital income taxation. In this article, we consider an overlapping generations (OLG) model with two
ability types and asymmetric information between the private sector and the government, that is, an extension of the static two-type optimal income tax model developed by Stern (1982) and Stiglitz (1982). The set of tax instruments consists of nonlinear taxes on labor income and capital income. Therefore, the tax instruments considered here are based on informational limitations and not on any other a priori restrictions. The overall purpose is to analyze how the appearance of positional preferences modifies the optimal income tax structure by comparison with the outcome of the standard two-type OLG model where people only care about their absolute consumption levels.

Only a few earlier studies have dealt with optimal nonlinear taxation in economies where people have positional preferences. To our knowledge, the first was a paper by Oswald (1983), who assumes a continuous ability distribution and that each individual compares his/her own consumption with a reference point; the latter is interpretable as reflecting either jealousy or altruism. Oswald shows that allowing for jealousy/altruism affects the optimal tax structure in a complex way and that several standard results of optimal tax theory (such as zero marginal tax rates at the ends of the skill distribution and that differentiated commodity taxes should not be used with certain forms of separable preferences) may no longer apply. Furthermore, the results show that if the utility function is separable in the measure of reference consumption, then the marginal tax rates are higher in an economy with predominantly jealous people and lower in an economy with predominantly altruistic people compared with the standard model without social interaction. Tuomala (1990) uses a similar model where the utility of each individual depends negatively on the average consumption of others and generalizes some findings by Oswald beyond additive separability. In addition, he provides numerical simulations showing, for instance, that the optimal marginal tax rates may be substantially higher when taking positional concerns into account. Ireland (2001) also uses a model with a continuous ability distribution and nonlinear taxation of labor income. He assumes that individuals signal their social status positions, which in turn necessitates using resources that could otherwise have been used for beneficial consumption. This constitutes an incentive for the government to intervene, meaning (again) that social interaction justifies the use of distortionary taxation. Finally, Aronsson and Johansson-Stenman (2008b) analyze a static two-type model where agents value their own consumption both in absolute terms and relative to a measure of reference consumption (the average consumption in the economy as a whole). The results show, among other things, how the redistributive and corrective roles of income taxation may interact, due to possible differences between agents with respect to the degree of positionality.

This article is also related to a small—yet growing—literature dealing with redistribution under asymmetric information in dynamic economies. The seminal contribution here is a paper by Ordover and Phelps (1979). In a model with a continuum of ability types, they show (among other things) that if leisure is separable from private consumption in terms of the utility function (so the marginal rate of substitution between present and future consumption does not depend on the leisure choice other than via income), then the marginal capital income tax rate should be zero for each ability type. Pirttilä and Tuomala (2001), in a generalization of the model in Brett (1997), consider an OLG model with two ability types. Their results show that production inefficiency at the second-best optimum (which is a consequence of the desire to relax the self-selection constraint) justifies capital income taxation, whereas the marginal labor income tax rates take the same general form as in Stiglitz (1982), that is, a positive marginal labor income tax rate should be imposed on the low-ability type and a negative marginal labor income tax rate on the high-ability type. A somewhat related argument for using capital income taxation is found by Aronsson et al. (2009); they show that the appearance of equilibrium unemployment may justify capital income taxation, as it implies intertemporal production inefficiency at the second-best optimum. Finally, Boadway et al. (2000) analyze nonlinear labor income taxation and proportional capital income taxation in a model where both ability and initial wealth are unobserved by the government. In their framework, the capital income tax is interpretable as an indirect instrument to tax wealth.
This study makes (at least) two distinct contributions to the literature. First, because we use a dynamic model we are able to consider capital income taxation. As far as we know, the only previous study that analyzes capital income taxation under relative consumption concerns is Abel (2005). He considers optimal capital income taxation in an OLG model where all consumers of a given generation are identical, and where a linear capital income tax constitutes the only tax instrument. This article, in contrast, analyzes the remaining role of capital income taxation when the labor income tax has been chosen in an optimal way. As earlier research indicates that the capital income tax might be a useful tool for relaxing the self-selection constraint, as noted earlier, a natural question is whether this tax is also useful for purposes of internalizing positional externalities. We show (for a special case) that under plausible empirical estimates regarding relative consumption concerns, the marginal capital income tax rate implemented for the low-ability type may be substantially smaller in absolute value than would be predicted by a model without positional concerns. In addition, the well-known result of zero marginal capital income tax rates under leisure separability (Ordover and Phelps, 1979) generalizes to our more general framework for a natural benchmark case.

Second, our article recognizes—and incorporates into the analysis—the idea that each individual may compare himself/herself more with some people than with others, that is, the appropriate measure of “reference consumption” at the individual level need not necessarily be based on the average consumption in the economy as a whole (which is the common assumption in earlier literature on public policy in economies where agents have positional preferences). For example, some evidence suggests that people compare themselves more with people who are similar to themselves (Runciman, 1966), whereas McBride (2001) found that people’s well-being depends on their income relative to the income of people belonging to the same generation as themselves. Others, such as Veblen (1899), Duesenberry (1949), and Schor (1998), have argued for the importance of an asymmetry, such that “low-income groups are affected by consumption of high-income groups but not vice versa” (Duesenberry, 1949, p. 101). This is also consistent with the empirical findings of Bowles and Park (2005) that more inequality in society tends to imply longer work hours. Therefore, in addition to measures based on the average consumption in the economy as a whole (which is our reference case), we consider two alternative approaches for measuring reference consumption at the individual level: the average consumption among people in the same generation and the average consumption among high-ability individuals.

The outline of the study is as follows: Section 2 presents the model and the outcome of private optimization based on the assumption that each individual compares his/her consumption with the average consumption in that period. Section 3 characterizes the corresponding optimal tax problem of the government, whereas Sections 4 presents the optimal labor income and capital income taxation results in a format that aims to facilitate straightforward interpretations and comparisons with earlier literature. Section 5 presents results for alternative reference points: in Subsection 5.1, we analyze the case where people solely make within-generation consumption comparisons, and in Subsection 5.2, we analyze the case where the reference point solely depends on the consumption by high-ability individuals. Section 6 summarizes and concludes the article, whereas proofs are presented in the Appendix.

2. POSITIONAL PREFERENCES, FIRMS, AND MARKET EQUILIBRIUM

2.1. The OLG Framework and Utility Functions. Consider an OLG model where each agent lives for two periods. Following the convention in earlier literature, we assume that each individual works during the first period of life and does not work during the second. There are two types of individuals, where the low-ability type (type 1) is less productive than the high-ability type (type 2). The number of individuals of ability-type \(i\) who were born at the beginning of period \(t\) is denoted \(n_i^t\). Each such individual cares about his/her consumption when young and

5 In Abel’s study, the tax revenues are returned lump-sum to the old generation. The model also contains a social security system (based on lump-sum payments) with its own budget.
when old, $c_t^i$ and $x_{t+1}^i$, and his/her leisure when young, $z_t^i$, given by a time endowment, $H$, less the hours of work, $l_t^i$ (when old, all available time is leisure). In addition, as the agents are assumed to have positional preferences, they also compare their own consumption with a measure of reference consumption. We follow earlier comparable literature in assuming that the private consumption good (the consumption of which is denoted $c$ when young and $x$ when old) is in part a positional good, whereas leisure is completely nonpositional.\(^6\)

The preferences for relative consumption, or positional preferences, can of course still be modeled in many different ways. Here we follow the approach chosen by many earlier studies by letting the relative consumption be described by the difference between an individual’s own consumption and the average consumption in the economy as a whole, given by $\bar{c}_t$ at time $t$; cf., for example, Akerlof (1997), Corneo and Jeanne (1997), Ljungqvist and Uhlig (2000), Bowles and Park (2005), and Carlsson et al. (2007).\(^7\) The utility function of ability-type $i$ born in the beginning of period $t$ can then be written as

\[
U_t^i = v_t^i(c_t^i, z_t^i, x_{t+1}^i, c_t^i - \bar{c}_t, x_{t+1}^i - \bar{c}_{t+1}) = u_t^i(c_t^i, z_t^i, x_{t+1}^i, \bar{c}_t, \bar{c}_{t+1}).
\]

The utility function $v_t^i(\cdot)$ is increasing in each argument, implying that $u_t^i(\cdot)$ is decreasing in $\bar{c}_t$ and $\bar{c}_{t+1}$ and increasing in the other arguments. Both $v_t^i(\cdot)$ and $u_t^i(\cdot)$ are assumed to be twice continuously differentiable in their respective arguments and strictly concave. The level of reference consumption in period $t$ is measured by the average consumption among all people alive in this time period:

\[
\bar{c}_t = \frac{n_t^1 c_t^1 + n_t^2 c_t^2 + n_{t-1}^1 x_t^1 + n_{t-1}^2 x_t^2}{N_t},
\]

in which $N_t = n_t^1 + n_t^2 + n_{t-1}^1 + n_{t-1}^2$. This means that each individual compares his/her own consumption with the average consumption in each period. We also assume that each individual treats the reference levels, $\bar{c}_t$ and $\bar{c}_{t+1}$, as exogenous.

The utility function in Equation (1) is quite general and may vary both between ability types and across generations, and is furthermore not necessarily time separable, meaning, for example, that the marginal rate of substitution between relative and absolute consumption when old is not necessarily independent of the consumption level when young. Thus, the model is flexible enough to encompass habit formation in private consumption. We will perform much of the analysis by using the second—and more general—utility formulation in Equation (1), that is, the function $u_t^i(\cdot)$. This case resembles a classical externality problem, for example, in terms of pollution associated with private consumption. However, we will need the first utility formulation based on the function $v_t^i(\cdot)$ when we relate the optimal tax policy to the extent that people care about relative consumption. The definition of such measures is the issue to which we turn next.

2.2. The Degree of Consumption Positionality. Since much of the subsequent analysis is focused on relative consumption concerns, it is useful to introduce measures of the degree to which such concerns matter for each individual. By defining $\Delta_t^{i,c} = c_t^i - \bar{c}_t$ and $\Delta_t^{i,x} = x_{t+1}^i - \bar{c}_{t+1}$, we can rewrite the first part of Equation (1) as

\[
U_t^i = v_t^i(c_t^i, z_t^i, x_{t+1}^i, \Delta_t^{i,c}, \Delta_t^{i,x}).
\]

\(^6\) As noted by Aronsson and Johansson-Stenman (2008b), it is of course possible to extend the analysis by allowing people to care about their relative amount of leisure. We leave this to future research. Our conjecture is that the major qualitative insights will still hold as long as private consumption is more positional than leisure, which is consistent with the limited empirical evidence (Solnick and Hemenway, 1998, 2005; Carlsson et al., 2007).

\(^7\) Alternative approaches include ratio comparisons (Boskin and Sheshinski, 1978; Layard, 1980; Abel, 2005; Wendner and Goulder, 2008) and comparisons of ordinal rank (Frank, 1985; Hopkins and Kornienko, 2004). Dupor and Liu (2003) consider a specific flexible functional form that includes the difference comparison and ratio comparison approaches as special cases.
We can then define the degree of consumption positionality (cf., e.g., Johansson-Stenman et al., 2002; Aronsson and Johansson-Stenman, 2008b) when young and old, respectively, based on the utility function in Equation (1) as follows:

\begin{align}
\alpha^j_{t,c} &= \frac{v^j_{t,\Delta t} + v^j_{t,c}}{v^j_{t,\Delta t} + v^j_{t,c}}, \\
\alpha^j_{t+1,x} &= \frac{v^j_{t+1,\Delta t} + v^j_{t+1,x}}{v^j_{t+1,\Delta t} + v^j_{t+1,x}},
\end{align}

where \( v^j_{t,c} \equiv \partial v^j_t / \partial c^j_t \) and similarly for the other variables. The term \( \alpha^j_{t,c} \) can then be interpreted as the fraction of the overall utility increase from the last dollar spent in period \( t \), that is, when young, that is due to the increased relative consumption. For instance, if \( \alpha^j_{t,c} \) approaches zero, then relative consumption does not matter on the margin, whereas in the other extreme case where \( \alpha^j_{t,c} \) approaches one, absolute consumption does not matter (i.e., all that matters is relative consumption). The interpretation of \( \alpha^j_{t+1,x} \) is analogous except that this term reflects the degree of consumption positionality when old instead of when young. From the assumptions about the utility functions, we have \( 0 < \alpha^j_{t,c}, \alpha^j_{t+1,x} < 1 \). In addition, let us denote the average degree of consumption positionality in period \( t \)

\[ \bar{\alpha}_t = \sum_i \alpha^i_{t,x} \frac{n^i_{t-1}}{N_t} + \sum_i \alpha^i_{t,c} \frac{n^i_t}{N_t} \in (0, 1). \]

In other words, \( \bar{\alpha}_t \) reflects the average value of the degree of consumption positionality among the people alive in period \( t \).

2.3. Individual Optimization and Market Equilibrium. The individual budget constraint is given by

\begin{align}
(5a) \quad w^j_t l^j_t - T_t(w^j_t l^j_t) - s^j_t &= c^j_t, \\
(5b) \quad s^j_t (1 + r_{t+1}) - \Phi_{t+1} (s^j_t r_{t+1}) &= x^j_{t+1},
\end{align}

where \( s^j_t \) is savings, \( r_{t+1} \) is the market interest rate, and \( T_t(\cdot) \) and \( \Phi_{t+1}(\cdot) \) denote the payments of labor income and capital income taxes, respectively. The first-order conditions for the hours of work and savings can be written as

\begin{align}
(6) \quad u^j_{t,c} w^j_t [1 - T_t(w^j_t l^j_t)] - u^j_{t,z} &= 0, \\
(7) \quad -u^j_{t,c} + u^j_{t,x} [1 + r_{t+1} (1 - \Phi'_{t+1}(s^j_t r_{t+1}))] &= 0,
\end{align}

in which \( u^j_t = \partial u^j_t / \partial c^j_t \), \( u^j_{t,z} = \partial u^j_t / \partial z^j_t \) and \( u^j_{t,x} = \partial u^j_t / \partial x^j_{t+1} \), whereas \( T_t(w^j_t l^j_t) \) and \( \Phi'_{t+1}(s^j_t r_{t+1}) \) are the marginal labor income tax rate and the marginal capital income tax rate, respectively.

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As our model does not distinguish between different types of commodities, we abstract from commodity taxation throughout the article. This approach has also been taken in most earlier comparable literature (see Section 1). This does not reflect a belief that commodity taxation is unimportant in connection to positional preferences. However, there are several practical problems associated with such extensions. For example, different variants of the same group of commodities, such as cars, may be characterized by very different degrees of positionalität. Moreover, the theoretical analysis would become considerably more complex, suggesting that commodity taxation warrants a paper of its own.
The production sector consists of identical competitive firms producing a homogenous good with constant returns to scale. Given these characteristics, the number of firms is not important and will be normalized to one for notational convenience. The production function is given by

\[ F(L_i^1, L_i^2, K_i; t) = g(\theta^1 L_i^1 + \theta^2 L_i^2, K_i; t), \]

where \( L_i^1 = n_i^1 l_i^1 \) is the total number of hours of work supplied by ability-type \( i \) in period \( t \), and \( K_i \) is the capital stock in period \( t \); \( \theta^1 \) and \( \theta^2 \) are positive constants. The direct time dependency implies that we allow for exogenous technological change, that is, productivity improvements. The firm obeys the necessary optimality conditions

\[ F_L(L_i^1, L_i^2, K_i; t) = \frac{\partial g}{\partial (\theta^1 L_i^1 + \theta^2 L_i^2)} \theta^i = w_i^i \text{ for } i = 1, 2, \]

\[ F_K(L_i^1, L_i^2, K_i; t) = \frac{\partial g}{\partial K_i} = r_i. \]

Equation (9), for \( i = 1, 2 \), implies that the wage ratio, that is, relative wage rate, is constant both within each period and over time.\(^9\)

3. THE GOVERNMENT’S DECISION PROBLEM

3.1. Objective and Constraints. We assume that the government faces a general social welfare function as follows:

\[ W = W(n_0^1 U_0^1, n_0^2 U_0^2, n_1^1 U_1^1, n_1^2 U_1^2, \ldots), \]

which is increasing in each argument. Since the optimum conditions are expressed for any such social welfare function, they are necessary optimum conditions for a Pareto efficient allocation.\(^10\) A similar formulation is used by Pirtilä and Tuomala (2001), although they in addition assume that the social welfare function is utilitarian within each generation.

The informational assumptions are conventional. The government is able to observe income, although ability is private information. As in most earlier literature on the self-selection approach to optimal taxation, we assume that the government wants to redistribute from the high-ability to the low-ability type.\(^11\) This means that the most interesting aspect of self-selection is to prevent the high-ability type from pretending to be a low-ability type. The self-selection constraint that may bind then becomes

\[ U_i^2 = u_i^2(c_i^2, z_i^2, x_{i+1}^2, \bar{c}_i, \bar{c}_{i+1}) \geq u_i^2(c_i^1, H - \phi l_i^1, x_{i+1}^1, \bar{c}_i, \bar{c}_{i+1}) = \bar{U}_i^2, \]

where \( \phi = w_i^1 / w_i^2 = \theta^1 / \theta^2 \) is the wage ratio, which is a constant by the assumptions made earlier. The expression on the right-hand side of the weak inequality in (12) is the utility of the mimicker; in what follows, a caret (\(^\wedge\)) above a variable refers to the mimicker. Although the mimicker enjoys

\(^9\) This simplifying assumption is made solely for analytical convenience, as endogenous relative wage rates are not particularly important for the qualitative results derived below (i.e., for how the appearance of positional preferences affects the optimal marginal labor income and capital income tax rates). Readers interested in the more general case with endogenous relative wage rates are referred to the background working paper by Aronsson and Johansson-Stenman (2008a).

\(^10\) All results obtained here that are independent of the social welfare function (i.e., basically all results that we comment on) could have been obtained by instead explicitly solving for the Pareto efficient allocation by maximizing the utility of one ability type born in a certain period while holding the utility constant for all other agents (the other ability type born in the same period and both ability types born in all other periods). The chosen strategy is motivated by convenience, as it simplifies the presentation.

\(^11\) This of course implies restrictions on the social welfare function beyond what is stated earlier.
the same consumption as the low-ability type in each period, he/she enjoys more leisure (as the mimicker is more productive than the low-ability type).¹²

Note that $T_i(\cdot)$ is a general labor income tax, which can be used to implement any desired combination of $l_1^t, c_1^t, l_2^t,$ and $c_2^t,$ given the savings chosen by each ability type. Therefore, we will use $l_1^t, c_1^t, l_2^t,$ and $c_2^t,$ instead of the parameters of the labor income tax function, as direct decision variables in the optimal tax problem. Note also that the general capital income tax, $\Phi_{t+1}(\cdot),$ can be used to implement any desired combination of $c_1^t, x_{t+1}^1, c_2^t, x_{t+1}^2,$ and $K_{t+1},$ given the labor income of each individual. Therefore, instead of choosing the parameters of the capital income tax function directly, we formulate the optimization problem such that $x_{t+1}^1, x_{t+1}^2,$ and $K_{t+1}$ are also used as direct decision variables. The resource constraint is given by

\begin{equation}
F(L_1^t, L_2^t, K_t; t) + K_t - \sum_{i=1}^2 [n_i^t c_i^t + n_{i-1}^t x_i^t] - K_{t+1} = 0. \tag{13}
\end{equation}

Equation (13) means that output is used for private consumption and net investments. Equations (12) and (13) together constitute the set of restrictions facing the government. The Lagrangean can then be written as

\begin{equation}
L = W(n_0^1 U_0^1, n_0^2 U_0^2, n_1^1 U_1^1, n_1^2 U_1^2, \ldots) + \sum_{i} \lambda_{i} [U_i^2 - U_i^1] \\
+ \sum_{i} \gamma_{i} \left[ F(L_i^1, L_i^2, K_i; t) + K_i - \sum_{i=1}^2 [n_i^t c_i^t + n_{i-1}^t x_i^t] - K_{t+1} \right]. \tag{14}
\end{equation}

For further use, let $\hat{u}_i^2 = u_i^2(c_i^t, H - \phi l_1^t, x_{t+1}^1, \tilde{e}_i, \tilde{c}_{i+1})$ denote the utility of the mimicker based on the second utility formulation in Equation (1). As the decision problem facing the government is written,¹³ the direct decision variables relevant for generation $t$ are $l_1^t, c_1^t, x_{t+1}^1, l_2^t, c_2^t, x_{t+1}^2, K_t,$ and $K_{t+1}.$ The first-order conditions are presented in the Appendix.

### 3.2. The Positionality Effect

Let us now turn to the welfare effect of an increase in the reference consumption, which will play a key role for marginal income taxation. The derivative of the Lagrangean with respect to $\tilde{c}_i$ can be written as

\begin{equation}
\frac{\partial L}{\partial \tilde{c}_i} = \sum_{i=1}^2 \frac{\partial W}{\partial (n_{i-1}^t U_i^t)} n_{i-1}^t u_{i-1, \tilde{c}_i} + \sum_{i=1}^2 \frac{\partial W}{\partial (n_i^t U_i^t)} n_i^t u_i^t, \tilde{c}_i \\
+ \lambda_{i-1} [u_{i-1, \tilde{c}_i} - \hat{u}_{i-1, \tilde{c}_i}] + \lambda_i [u_{i, \tilde{c}_i} - \hat{u}_{i, \tilde{c}_i}]. \tag{15}
\end{equation}

We will refer to this derivative as measuring the positionality effect in period $t,$ since it reflects the overall welfare effects of a change in the level of reference consumption in period $t,$ ceteris paribus. By using the first utility formulation in Equation (1), that is, the function $\nu_i^t(\cdot),$ this effect

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¹² Given the set of available policy instruments in our framework, it is possible for the government to control the present and future consumption as well as the hours of work of each ability type (this is discussed more thoroughly later). As a consequence, in order to be a mimicker, the high-ability type must mimic the point chosen by the low-ability type on each tax function (both the labor income tax and the capital income tax), and thus consume equally much in both periods.

¹³ Note that there is a potential time inconsistency problem involved here since the government may have incentives to modify the second period taxation facing each generation once the individuals have revealed their true types. Although we acknowledge this potential problem, we follow earlier comparable literature by only considering situations where the government commits to its tax policy. This is motivated by the observation that lack of commitment from the point of view of the government opens a spectrum of possibilities for modeling both public policy and the response by the private sector, which would be beyond the scope of this article.
can be rewritten in terms of the individual degrees of consumption positionality. Let us use the short notation

$$\Gamma_t = \frac{\lambda_{t-1} \bar{u}_{t-1,x}^2}{\gamma_t N_t} [\hat{\alpha}_t^{2,x} - \alpha_t^{1,x}] + \frac{\lambda_t \bar{u}_{t,c}^2}{\gamma_t N_t} [\hat{\alpha}_t^{2,c} - \alpha_t^{1,c}]$$

for the positionality differences between the mimicker and the low-ability type (measured for both generations alive in period t), where $\Gamma_t > 0 (< 0)$ if the mimicker is always, that is, as both young and old, more (less) positional than the low-ability type. We can then derive the following result:

**Lemma 1.** The welfare effect of increased reference consumption in period t can be written as

$$\frac{\partial L}{\partial \bar{c}_t} = N_t \gamma_t \left( \Gamma_t - \bar{\alpha}_t \right) = -\frac{\bar{\alpha}_t \gamma_t N_t}{1 - \bar{\alpha}_t} + \frac{1}{1 - \bar{\alpha}_t} \left[ \lambda_{t-1} \bar{u}_{t-1,x}^2 [\hat{\alpha}_t^{2,x} - \alpha_t^{1,x}] + \lambda_t \bar{u}_{t,c}^2 [\hat{\alpha}_t^{2,c} - \alpha_t^{1,c}] \right].$$

Therefore, increased reference consumption in period t reduces the welfare, so $\partial L / \partial \bar{c}_t < 0$, if and only if $\Gamma_t < \bar{\alpha}_t$. A sufficient condition for this to hold is that $\alpha_t^{1,c} \geq \hat{\alpha}_t^{2,c}$ and $\alpha_t^{1,x} \geq \hat{\alpha}_t^{2,x}$, meaning that the young and old low-ability types, respectively, are at least as positional as the corresponding mimicker in period t.

Two mechanisms are worth noticing. First, in the absence of the self-selection constraint, that is, if ability-type specific lump-sum taxes were possible to implement, an increase in the reference consumption would unambiguously decrease the welfare, since the reference consumption enters the utility function of each individual via the arguments $\Delta_t^{l,c}$ and $\Delta_t^{l,x} = \Delta_t^{l,x} - \bar{\Delta}_t^{l,x}$. Thus, the reference consumption constitutes a negative externality for each utility type in each period. This explains the effect of $\bar{\alpha}_t$ in Equation (16), which relates the positionality effect to the average degree of positionality without any reference to differences in the degree of positionality between ability types. Second, if the low-ability type is more positional than the mimicker in both generations alive in period t (i.e., generations t and t−1), then an increase in the reference consumption means a larger utility loss for the low-ability type than for the mimicker; as such, it contributes to an additional welfare loss via the self-selection constraint. However, if the mimicker is more positional than the low-ability type, then an increase in the reference consumption contributes to relax the self-selection constraint, implying that $\Gamma_t > 0$ in Equation (16); this mechanism will be discussed in more detail subsequently. In this case, the sign of $\partial L / \partial \bar{c}_t$ can be either positive or negative depending on whether or not $\Gamma_t < \bar{\alpha}_t$.

### 4. Results

In this section, we present the optimality conditions for the marginal labor income tax rates and the marginal capital income tax rates in a format that facilitates straightforward economic interpretations, as well as comparisons with the standard optimal income tax model in which relative consumption concerns are absent. Note that the expressions for the marginal income tax rates presented below are valid in each time period along the general equilibrium path, that is, for all t. The reason is, of course, that the second best optimal resource allocation means that the first order conditions used to derive these expressions are fulfilled in each time period. Moreover, the formulas for marginal income taxation hold regardless of the sign and magnitude of individual responses to a change in the reference consumption. In other words, they hold regardless of whether or not individual consumption increases as a result of an increase in the average consumption (or reference consumption more generally) in the same period, i.e., regardless of whether there is in this sense a “keeping-up-with-the-Joneses consumption effect” and, therefore, also regardless of whether such possible effects are small or large.¹⁴

¹⁴ See, for example, Arrow and Dasgupta (2009) for some theoretical analysis of such effects in an intertemporal model.
4.1. Labor Income Taxation. By defining the marginal rate of substitution between leisure and private consumption for ability-type \( i \) as

\[
MRS_{z,c}^{i,t} = \frac{u_{i,z}^{t}}{u_{i,c}^{t}},
\]

and similarly for the mimicker, we obtain the following expressions for the marginal labor income tax rates by using the government’s first-order conditions for \( l_{1}^{t}, c_{1}^{t}, l_{2}^{t}, \) and \( c_{2}^{t} \) together with Equation (6):

\[
T_{i}'(w_{i}^{t}l_{1}^{t}) = \frac{\lambda_{i}^{*}}{w_{i}^{t}n_{i}^{t}} [MRS_{z,c}^{1,t} - \phi MRS_{z,c}^{2,t}] - \frac{MRS_{z,c}^{1,t}}{\gamma_{i} w_{i}^{t} N_{i}^{t}} \frac{\partial \mathcal{L}}{\partial \bar{c}_{i}^{t}},
\]

(17)

\[
T_{i}'(w_{i}^{t}l_{2}^{t}) = -\frac{MRS_{z,c}^{2,t}}{\gamma_{i} w_{i}^{t} N_{i}^{t}} \frac{\partial \mathcal{L}}{\partial \bar{c}_{i}^{t}},
\]

(18)

where \( \lambda_{i}^{*} = \lambda_{i}^{2} u_{i,c}^{2} / \gamma_{i} \). The marginal labor income tax rates in Equations (17) and (18) are straightforward extensions of the results derived in a static model by Aronsson and Johansson-Stenman (2008b). However, the present model is more general in the sense of capturing that a change in the reference consumption in period \( t \), induced by a change in the hours of work supplied by the young generation, will affect the well-being of both the young and the old generation in that period. This will be discussed more thoroughly later.

The first term of Equation (17) is analogous to results derived in earlier literature and is due to the self-selection constraint. With \( MRS_{z,c}^{1,t} > MRS_{z,c}^{2,t} \) (which applies if the preferences do not differ between ability types), the contribution of the self-selection constraint is to increase the marginal labor income tax rate of the low-ability type; this effect is well understood from earlier research (Stiglitz, 1982).

The final part of each formula reflects the relative consumption concerns. By combining Lemma 1 with Equations (17) and (18), we obtain the following result.

**Proposition 1.** If the young and old low-ability types, respectively, are at least as positional as the corresponding mimicker in period \( t \), or if the positionality differences are sufficiently small so that \( \Gamma_{t} < \bar{a}_{i} \), then the positionality effect contributes to increase the marginal labor income tax rate facing each ability type in period \( t \), ceteris paribus.

Note that the positionality effect discussed in Proposition 1 contains two parts: an externality-correcting component and a component that serves to relax the self-selection constraint. In order to see this more clearly, we will combine Equations (16)–(18) in order to decompose the positionality effect. Let us use the short notations \( \sigma_{i}^{1} \) and \( \sigma_{i}^{2} \) for the optimal marginal labor income tax rates without relative consumption concerns, where

\[
\sigma_{i}^{1} = \frac{\lambda_{i}^{*}}{w_{i}^{t} n_{i}^{t}} [MRS_{z,c}^{1,t} - \phi MRS_{z,c}^{2,t}] \quad \text{and} \quad \sigma_{i}^{2} = 0.
\]

We can then rewrite the formulas for the marginal labor income tax rates such that the contribution of positionality is decomposed into two effects as follows.

**Proposition 2.** The optimal marginal labor income tax rate for each ability type can be written in the following additive form (for \( i = 1, 2 \)):

\[
T_{i}'(w_{i}^{t}l_{i}^{t}) = \sigma_{i}^{1} + \left[ 1 - \sigma_{i}^{1} \right] \bar{a}_{i} - \left[ 1 - \sigma_{i}^{1} \right] [1 - \bar{a}_{i}] \frac{\Gamma_{i}}{1 - \Gamma_{i}}.
\]

(19)

Equation (19) is an intertemporal analogue to (and has the same general interpretation as) a corresponding tax formula derived by Aronsson and Johansson-Stenman (2008b) in a static
model; the difference is that $\tilde{\alpha}_{t}$ and $\Gamma_{t}$ here reflect the preferences of both generations alive in period $t$. The intuition is that the young generation, which is the only generation working and paying labor income taxes, also imposes positional externalities on the old generation. The necessary additional correction (to internalize the positional externality that the young generation imposes on the old) is, therefore, part of the marginal labor income tax rates.

To interpret Equation (19), note first that in the special case where the resource allocation is first best, meaning that $\lambda_{t} = 0$ for all $t$, we have $\sigma_{t}^{1} = \sigma_{t}^{2} = \Gamma_{t} = 0$, so $T_{t}(w_{1}^{1}l_{1}^{1}) = T_{t}(w_{2}^{2}l_{2}^{2}) = \tilde{\alpha}_{t}$, which exemplifies a straightforward Pigouvian tax. The interpretation is that each individual is taxed for the negative positional externality that he/she imposes on other people.

Returning to our more general second-best model, the intuition is straightforward. The first term on the right-hand side of Equation (19) is the tax expression that would follow without any positional concern. The second term measures the marginal external cost of consumption—as reflected by the average degree of positionality—although its contribution to the marginal labor income tax rates is modified by comparison with the first-best. Increased private consumption, associated with an increase in hours of work, causes negative external costs here as well; for the low-ability type, however, these external costs are smaller than in the first-best provided that $\sigma_{t}^{1} > 0$ (which is the case we discussed earlier). The intuition is that the fraction of an income increase that is already taxed away does not give rise to positional externalities.

The third term on the right-hand side of Equation (19) reflects self-selection effects of positional concerns. Suppose first that $\Gamma_{t} > 0$, in which case the mimicker is more positional than the low-ability type. This means that increased reference consumption gives rise to a larger utility loss for the mimicker than it does for the low-ability type. Therefore, the government may relax the self-selection constraint by implementing policies that lead to increased reference consumption. This provides an incentive for the government to implement a lower marginal labor income tax rate than it would otherwise have done, which means that the third term contributes to decrease the marginal labor income tax rate. Consequently, if $\Gamma_{t}$ is positive and sufficiently large, then this effect may (at least theoretically) dominate the externality-correcting component, implying that relative consumption concerns contribute to reduce the marginal labor income tax rates. If on the other hand $\Gamma_{t} < 0$, then the opposite argument applies. The latter case also explains in greater detail why the positionality effect unambiguously contributes to increase the marginal labor income tax rates if the young and old low-ability types, respectively, are at least as positional as the corresponding mimicker.

Let us briefly discuss how the appearance of positional preferences may affect the marginal labor income tax rates according to the empirical evidence. Consider first the high-ability type for whom $\sigma_{t}^{2} = 0$ and suppose also that $\Gamma_{t} = 0$. The latter is motivated by the lack of empirical evidence of how the degree of positionality varies with leisure, for a given consumption level (remember that the mimicker and the low-ability type have the same consumption levels). In this case, the optimal tax is equal to $\tilde{\alpha}_{t}$ for the high-ability type. According to the survey-experimental evidence of Alpizar et al. (2005) and Carlsson et al. (2007), $\tilde{\alpha}_{t}$ is in the order of magnitude of 0.5. Wendner and Goulder (2008) provide a rigorous treatment of existing survey-experimental evidence (see their Table 1, p. 1978), where they apply the same statistical methods to different data sets. They conclude that the evidence taken together suggests that $\tilde{\alpha}_{t}$ should be between 0.2 and 0.4. On the other hand, $\tilde{\alpha}_{t} = 0.8$ would be more in line with the evidence based on the happiness studies of Easterlin (1995) and Luttmer (2005). These are clearly very dramatic differences compared to the zero marginal tax rate that would apply in the absence of positional preferences. For the low-ability type, the same assumptions would imply corresponding effects, although the changes in relative terms would seem less dramatic, since the pure self-selection component, $\sigma_{t}^{1}$, is most likely positive (see above and Stiglitz, 1982).

4.2. Capital Income Taxation. Let us now turn to the marginal capital income tax structure. Define the marginal rate of substitution between consumption in periods $t$ and $t+1$ for ability-type $i$
and similarly for the mimicker. The optimal marginal capital income tax rates in period \( t + 1 \) are obtained by combining the government’s first order conditions for \( c^i_t \) and \( x^i_{t+1} \), for \( i = 1, 2 \), with the private first-order condition for saving in period \( t \):

\[
\Phi'_{t+1}(x^1_{t+1}) = \frac{\lambda_i \hat{u}_{i,x}}{\gamma_{t+1} n^1_{t+1}} [MRS^1_{c,x} - MRS^2_{c,x}]
+ \frac{1}{\gamma_{t+1} n^1_{t+1}} \left[ \frac{\partial \mathcal{L}}{\partial \hat{c}_t} \frac{1}{N_t} - \frac{\partial \mathcal{L}}{\partial \hat{c}_{t+1}} \frac{1}{N_{t+1}} \right],
\]

\[
\Phi'_{t+1}(x^2_{t+1}) = \frac{1}{\gamma_{t+1} n^1_{t+1}} \left[ \frac{\partial \mathcal{L}}{\partial \hat{c}_t} \frac{1}{N_t} - \frac{\partial \mathcal{L}}{\partial \hat{c}_{t+1}} \frac{1}{N_{t+1}} \right].
\]

Let us start by discussing the marginal capital income tax rate of the low-ability type. The first term on the right-hand side of Equation (20) is due to the self-selection constraint. It means that if the marginal rate of substitution between present and future consumption by the low-ability type exceeds (falls short of) the corresponding marginal rate of substitution faced by the mimicker, there is an incentive for the government to stimulate (discourage) the current consumption via a higher (lower) marginal capital income tax rate. As such, this incentive effect serves to relax the self-selection constraint by making mimicking less attractive (see also, e.g., Brett, 1997, and Pirttilä and Tuomala, 2001).

The second term, which takes the same general form as the expression for the marginal capital income tax rate facing the high-ability type, is novel and refers to the assumption that a private consumption good is, in part, a positional good. As the marginal capital income tax rate facing the high-ability type, is novel and refers to the assumption that if the marginal rate of substitution between present and future consumption by the low-ability type exceeds (falls short of) the corresponding marginal rate of substitution faced by the mimicker, there is an incentive for the government to reduce the average consumption in period \( t \), which in turn stimulates savings and discourages consumption in period \( t \). By analogy, the positionality effect in period \( t + 1 \) contributes to decrease the marginal capital income tax rates in period \( t + 1 \), whereas the positionality effect in period \( t + 1 \) contributes to increase the marginal capital income tax rates in the same period, ceteris paribus.

The intuition behind Proposition 3 is straightforward. The positionality effect in period \( t \) means that an increase in the reference consumption in period \( t \) gives rise to a welfare loss. This provides an incentive for the government to choose lower marginal capital income tax rates than it would otherwise have done, which in turn stimulates savings and discourages consumption in period \( t \). By analogy, the positionality effect in period \( t + 1 \) means that an increase in the reference consumption in period \( t + 1 \) results in a welfare loss. As a consequence, there is an incentive for the government to reduce the average consumption in period \( t + 1 \), which means that the government chooses higher marginal capital income tax rates than it would otherwise have done. The relative sizes of these two effects determine whether the appearance of positional preferences constitutes an incentive to tax or subsidize the capital income at the margin, ceteris paribus.

Let us use the short notation \( \delta^1_t \) for the optimal marginal capital income tax rate that would follow in the absence of any positionality effects, that is,

\[
\delta^1_t = \frac{\lambda_i \hat{u}_{i,x}}{\gamma_{t+1} n^1_{t+1}} [MRS^1_{c,x} - MRS^2_{c,x}] \quad \text{and} \quad \delta^2_t = 0.
\]
Then, using the decomposition of the positionality effect given by Equation (16), we obtain

\begin{equation}
\Phi_{t+1}'(s_t^i r_{t+1}) = \delta_t \frac{1}{1 - \Gamma_{t+1}} \frac{1}{1 - \Gamma_{t+1}} \left[ \frac{1}{1 - \Gamma_{t+1}} \left( 1 + \frac{1}{r_{t+1}} \right) \left[ \frac{1 - \bar{\alpha}_{t+1} - \Gamma_{t+1}}{1 - \bar{\alpha}_{t+1}} - \frac{1 - \bar{\alpha}_{t+1} - \Gamma_{t+1}}{1 - \bar{\alpha}_{t+1}} - \frac{\alpha_{t+1} - \bar{\alpha}_{t+1}}{1 - \bar{\alpha}_{t+1}} \right] \right].
\end{equation}

In Equation (22), the first term on the right-hand side means that positional concerns may modify the traditional self-selection component in the tax formula for the low-ability type, which will be explained more thoroughly later. The sign of the second term is clearly positive (negative) if and only if the expression in square brackets is positive (negative), which is the case when the average net degree of positionality—that is, the average degree of positionality adjusted for the self-selection effect—increases (decreases) over time.\(^{15}\) The intuition is that future increases in the degree of positionality constitute incentives for the government to reduce consumption in the future, which it does by implementing a higher marginal capital income tax rate than it would otherwise have done.

In the special case where the average degree of positionality and the difference in the degree of positionality between the mimicker and the low-ability type are constant over time, Equation (22) can be simplified. This situation may arise either if the economy approaches a steady state\(^{16}\) or from functional form restrictions on the utility function beyond those presented in Section 2. Consider the following result, which is an implication of Equation (22).

**Proposition 4.** If the average degree of positionality as well as the positionality differences between the mimicker and the low-ability type remain constant over time, so \(\bar{\alpha}_{t+1} = \bar{\alpha}_t = \bar{\alpha}\) and \(\Gamma_{t+1} = \Gamma_t = \Gamma\), then the marginal capital income tax rates reduce to

\begin{equation}
\Phi_{t+1}'(s_t^1 r_{t+1}) = \frac{\lambda_t t^2 x}{\gamma_t + \bar{\alpha}_t^2 r_{t+1}} \left[ MRS^{1,t}_{c,x} - \hat{MRS}^{2,t}_{c,x} \right] \frac{1 - \bar{\alpha}}{1 - \Gamma},
\end{equation}

\begin{equation}
\Phi_{t+1}'(s_t^2 r_{t+1}) = 0.
\end{equation}

Note that there is no direct effect of positionality in these tax formulas that is independent of other tax incentives. Therefore, in this special case, the appearance of positionality does not change the way we measure the marginal capital income tax rate of the high-ability type (compared to an economy without positional goods). The intuition is that under the conditions in the proposition, the current and future aspects of positionality cancel each other out to a large extent, suggesting that the incentives underlying capital formation are similar to those that would apply in economies without positional goods. However, this does not mean that the effect of positionality that still remains for the low-ability type is unimportant.

As long as leisure is not weakly separable from private consumption, the low-ability type and the mimicker will differ with respect to the marginal rate of substitution between present and future consumption. The contribution of this difference to the marginal capital income tax rate of the low-ability type is still affected by concern for positionality. In order to interpret the “positionality-weight” \([1 - \bar{\alpha}]/[1 - \Gamma]\), consider first the situation where \(MRS^{1,t}_{c,x} > \hat{MRS}^{2,t}_{c,x}\), meaning that the first term on the right-hand side of Equation (23) contributes to increase the marginal capital income tax rate of the low-ability type. As such, this term works to increase the current (first period) consumption of the low-ability type and, as a consequence, also the reference consumption in period \(t\). The expression \(1 - \bar{\alpha}\) serves to modify this effect, as increased reference consumption gives rise to positional externalities. In other words, if we (for the moment) were to abstract from differences in the degree of positionality between the mimicker and the low-ability type, implying that \(\Gamma = 0\), then the positionality weight would work to decrease

\(^{15}\) Strictly speaking, this result also presupposes that \(1 - \Gamma_{t+1} > 0\).

\(^{16}\) This requires that the preferences and technology do not change over time and that the economy approaches a stationary equilibrium in which \(l^i_t, c^i_t, x^i_t\) (for \(i = 1, 2\)) and \(K_t\) all remain constant over time.
the marginal capital income tax rate. This effect is counteracted (further strengthened) by \( \Gamma > 0 \) \(< 0 \), as increased reference consumption in this case relaxes (tightens) the self-selection constraint in period \( t \). The interpretation is analogous if \( MRS^{1,t}_{c,x} < MRS^{2,t}_{c,x} \), yet with the modification that the positionality weight then serves to adjust the effect that a capital subsidy has in terms of future consumption.

Let us also discuss the marginal capital income tax rates in the light of the empirical evidence regarding relative consumption concerns described above. For simplicity, let us first focus on the case illustrated in Proposition 4, in which positional concerns only affect the marginal capital income tax rate of the low-ability type. As with the marginal labor income tax rates, we concentrate the discussion on the contribution of the average degree of positionality by assuming that \( \Gamma = 0 \); the reason is again the lack of clear empirical evidence regarding differences in the degree of positionality across agent types. In this (highly simplified) case, Equation (23) suggests that the absolute value of the marginal capital income tax rate may be substantially smaller than would be predicted in the absence of positional concerns. In fact, with the expression proportional to \( 1 - \tilde{a} \) held constant,\(^{17} \) the positionality effect with \( \tilde{a} \) being between 0.2 and 0.8 contributes to scale down the absolute value of the marginal capital income tax rate by a factor between 1.2 and 5. Therefore, if \( MRS^{1,t}_{c,x} > MRS^{2,t}_{c,x} \), the relative consumption concerns imply an incentive for the government to adjust the capital income tax to discourage the current relative to the future consumption, a policy response reminiscent of the consumption tax discussed by Frank (1999, 2007, 2008). However, assume now instead that the degree of positionality increases with income, which is in line with some empirical evidence (Clark et al., 2008) and as argued already (1999, 2007, 2008). However, assume now instead that the degree of positionality increases with income, which is in line with some empirical evidence (Clark et al., 2008) and as argued already by Keynes (1930). A growing economy would then imply that the degree of positionality would also increase over time, which from Equation (22) ceteris paribus works in the direction of a positive marginal capital income tax rate for both ability types.

The following result is a direct consequence of Proposition 4.

**Corollary 1.** Suppose that the average degree of positionality as well as the positionality differences between the mimicker and the low-ability type remain constant over time. Then, if leisure is weakly separable from private consumption in the sense that \( U_i^t = q_i^t( f_i(c_i^t, x_i^{t+1}, \Delta_i^{t, \epsilon}, \Delta_i^{t+1, \epsilon}, z_i^t) \) describes the utility function, then both optimal marginal capital income tax rates are zero.

Note that although the function \( q_i^t(\cdot) \) may vary across ability types, the function \( f_i(\cdot) \) must be the same for both ability types. Although the above result is based on assumptions that may not seem entirely realistic, it is nevertheless interesting from the perspective of comparison with earlier literature. Corollary 1 implies that the important result derived by Ordover and Phelps (1979), for when capital income taxation is not needed, carries over to our more general case that includes relative consumption concerns.

To give a functional form example, note finally that the conditions underlying Corollary 1 would be fulfilled with a constant population growth rate together with a utility function given as follows (for \( i = 1, 2 \)):

\[
U_i^t = \frac{k^i \left[ c_i^t \left[ (1 - a)c_i^t + a \left[ c_i^t - \bar{c}_i^t \right] \right] \left[ (1 - b)x_i^{t+1} + b \left[ x_i^{t+1} - \bar{c}_i^{t+1} \right] \right] \right]^{1-\epsilon}}{1 - \epsilon^i}
\]

\[
= \frac{k^i \left[ c_i^t \left[ c_i^t - a\bar{c}_i^t \right] \left[ x_i^{t+1} - b\bar{c}_i^{t+1} \right] \right]^{1-\epsilon}}{1 - \epsilon^i},
\]

where \( k^i > 0 \) and \( \epsilon^i \in (0, 1) \) are ability-type specific parameters, and \( a \) and \( b \) are constants such that \( 0 < a, b < 1 \). This example does not presuppose that the economy has reached a steady state. Instead, it follows immediately from the formulation in the first row that the degree of positionality is equal to \( a \) when young and \( b \) when old for each ability type (and for the

\(^{17}\) In reality, positional concerns of course give rise to indirect effects on the other terms as well. Therefore, this discussion only refers to the direct influence of the positionality effect.
mimicker).\(^{18}\) It is also straightforward to show that \(MRS_{c,t}^{i,x}\) —when calculated on the basis of this utility function—does not directly depend on \(z_i^t\) so that \(MRS_{c,t}^{1,i} = MRS_{c,t}^{2,i}\) and, therefore, the marginal capital taxes are zero for both ability types.

5. ALTERNATIVE REFERENCE POINTS: WITHIN-GENERATION AND UPWARD SOCIAL COMPARISONS

So far, we have assumed that the average consumption in the economy as a whole constitutes the appropriate measure of reference consumption, which follows earlier comparable literature. However, what constitutes the most appropriate measure of reference consumption can be discussed, and one may argue that people compare themselves more with some people than with others. In particular, it is possible that people primarily compare their own consumption with the consumption by other people in their own generation. Alternatively, one may argue that people primarily compare themselves with high-income earners, as suggested by, for example, Duesenberry (1949). In Subsection 5.1, we consider the case where people make within-generation comparisons, whereas Subsection 5.2 analyzes the case where people solely compare themselves with high-income earners, as suggested by, for example, Duesenberry (1949). In Subsection 5.1, we consider the case where people make within-generation comparisons, whereas Subsection 5.2 analyzes the case where people solely compare themselves with the high-ability type. In all other respects, we again make the same assumptions, for example, with respect to the production sector and available policy instruments, as in the previous sections.\(^{19}\)

5.1. Within-Generation Consumption Comparisons. When people solely make within-generation consumption comparisons, the utility function of ability-type \(i\) born in the beginning of period \(t\) can be written as

\[
U_i^t = v_i^t(c_i^t, z_i^t, x_{t+1}^i, c_i^t - \bar{c}_{i,t}, x_{t+1}^i - \bar{x}_{t+1,i}) = v_i^t(c_i^t, z_i^t, x_{t+1}^i, \Delta_{c,t}^{i,cc}, \Delta_{x,t+1}^{i,xx})
\]

As before, the utility function \(v_i^t(\cdot)\) is increasing in each argument, implying that \(u_i^t(\cdot)\) is decreasing in each average consumption level; \(\bar{c}_{i,t} = [n_1^t c_1^t + n_2^t c_2^t]/[n_1^t + n_2^t]\) represents the average consumption in period \(t\) measured for the generation born in period \(t\), whereas the average consumption of the same generation when old in period \(t + 1\) is given by \(\bar{x}_{t+1,i} = [n_1^t x_1^{t+1} + n_2^t x_2^{t+1}]/[n_1^t + n_2^t]\). Based on Equation (25), we can calculate within-generation measures of positionality as follows:

\[
\alpha_{t,cc}^{i,cc} = \frac{v_i^t(c_i^t, \Delta_{c,t}^{i,cc})}{v_i^t},\]

\[
\alpha_{t+1,xx}^{i,xx} = \frac{v_i^t(c_i^t, \Delta_{x,t+1}^{i,xx})}{v_i^t},\]

where \(\alpha_{t,cc}^{i,cc}\) can be interpreted as the fraction of the overall utility increase from the last dollar spent by a young individual of ability-type \(i\) in period \(t\) that is due to the increased relative consumption compared to other young people in period \(t\), whereas \(\alpha_{t+1,xx}^{i,xx}\) reflects the corresponding fraction compared to other old people when old in period \(t + 1\).

\(^{18}\) Alternatively, the conditions in the corollary would be fulfilled irrespective of the population growth rate if the utility function takes the form

\[
U_i^t = \kappa\{[c_i^t - a_0]\{[x_{t+1}^i - a\bar{x}_{t+1,i}]\}^{-\phi},\]

so that the degree of positionality is equal to \(a\) for all.

\(^{19}\) One may also argue that people are likely to make consumption comparisons over time, e.g., with earlier generations; see the background working paper by Aronsson and Johansson-Stenman (2008a).
We will now have two positionality effects. The welfare effect of an increase in the average consumption among the young in period $t$, $\bar{c}_{i,t}$, can be written as

$$(27a) \quad \frac{\partial L}{\partial \bar{c}_{i,t}} = -\frac{\gamma_t n_t \bar{a}_{i,t}^{cc}}{1 - \bar{a}_{i,t}^{cc}} + \frac{\lambda_t \bar{u}_{i,t}^2}{1 - \bar{a}_{i,t}^{cc}} \left[ \bar{a}_{i,t}^{2,cc} - \alpha_{i,t}^{1,cc} \right],$$

whereas the corresponding effect of an increase in the average consumption among the old in period $t + 1$, $\bar{x}_{i+1,t}$, can be written as

$$(27b) \quad \frac{\partial L}{\partial \bar{x}_{i+1,t}} = -\frac{\gamma_{t+1} n_t \bar{a}_{i+1,t}^{xx}}{1 - \bar{a}_{i+1,t}^{xx}} + \frac{\lambda_t \bar{u}_{i,t}^2}{1 - \bar{a}_{i+1,t}^{xx}} \left[ \bar{a}_{i+1,t}^{2,xx} - \alpha_{i+1,t}^{1,xx} \right].$$

In Equations (27a) and (27b), $\bar{a}_{i,t}^{cc}$ and $\bar{a}_{i+1,t}^{xx}$ are within-generation measures of the average degree of positionality. Clearly, and by analogy to Equation (16), each positionality effect is negative unless the mimicker is sufficiently more positional than the low-ability type, that is, unless the self-selection effect has the opposite sign of, and dominates, the direct effect; cf. Lemma 1.

By using the short notation

$$\Gamma_i^c = \frac{\lambda_t \bar{u}_{i,t}^2}{\gamma_t n_t} \left[ \bar{a}_{i,t}^{2,cc} - \alpha_{i,t}^{1,cc} \right],$$

where $n_t = n_t^1 + n_t^2$, we can then write the optimal marginal labor income tax rate for ability-type $i$ (for $i = 1, 2$) in the same general form as before,

$$(28) \quad T_i^c (w_i^t l_i^t) = \sigma_i^c + \left[ 1 - \sigma_i^c \right] \bar{a}_{i,t}^{cc} - \left[ 1 - \sigma_i^c \right] \left[ 1 - \alpha_{i,t}^{cc} \right] \Gamma_i^c \frac{1}{1 - \Gamma_i^c}.$$  

The only differences between Equation (28) and the corresponding tax formula in subsection 4.1, that is, Equation (19), are (i) that the average degree of positionality is here replaced by the corresponding within-generation measure, and (ii) the difference in the degree of positionality between the mimicker and the low-ability type is here based on the within-generation measure. In other words, it is only the positionality of the young generation, that is, of those working, that matters for the marginal labor income tax rates. The reason is, of course, that each young individual only imposes positional consumption externalities on other individuals of the same generation. The interpretation of Equation (28) is otherwise analogous to that of Equation (19).

The optimal marginal capital income tax rates can be written as

$$(29) \quad \Phi_{t+1}^c(s_{i}^1 r_{t+1}) = \frac{\lambda_t \bar{u}_{i,t}^2}{\gamma_t + n_t^1 \gamma_{t+1}} \left[ MRS_{c,x}^{1,t} - MRS_{c,x}^{2,t} \right] + \frac{1}{\gamma_t + r_{t+1}} \left[ \frac{\partial L}{\partial \bar{c}_{i,t}} \frac{1}{n_t} - MRS_{c,x}^{1,t} \frac{\partial L}{\partial \bar{x}_{i+1,t}} \frac{1}{n_t} \right],$$

$$(30) \quad \Phi_{t+1}^c(s_{i}^2 r_{t+1}) = \frac{1}{\gamma_t + r_{t+1}} \left[ \frac{\partial L}{\partial \bar{c}_{i,t}} \frac{1}{n_t} - MRS_{c,x}^{2,t} \frac{\partial L}{\partial \bar{x}_{i+1,t}} \frac{1}{n_t} \right].$$

The main interpretations are again similar to the previously analyzed case, implying that Proposition 3 will hold here too. By using the within-generation positionality effects together with the short notation

$$\Gamma_{t+1}^x = \frac{\lambda_t \bar{u}_{i,t}^2}{\gamma_t + n_t} \left[ \bar{a}_{i,t}^{2,xx} - \alpha_{i,t}^{1,xx} \right],$$


we can derive the following result (for \( i = 1, 2 \)):

\[
\Phi_t' (s_t, r_{t+1}) = s_t \left[ \frac{1 - \bar{\alpha}_{t+1}^x}{1 - \Gamma_{t+1}^x} + \frac{1 - \bar{\alpha}_{t+1}^{xx}}{1 - \Gamma_{t+1}^{xx}} \right] + \frac{1}{r_{t+1}} \left[ \frac{\bar{\alpha}_{t+1}^{xx} - \Gamma_{t+1}^{xx}}{1 - \bar{\alpha}_{t+1}^{xx} - \Gamma_{t+1}^{xx}} \right].
\]

Equation (31) is analogous to Equation (22) with the exception that the measures of positionality in Equation (31) are within-generation measures. The second term on the right-hand side of Equation (31) is positive (negative) if and only if the average net degree of positionality, that is, the average degree of positionality adjusted for the self-selection effect, is larger (smaller) when old than when young. Intuitively, a high degree of positionality means a larger social waste from consumption. Therefore, if the old are less positional than the young, which seems reasonable from an evolutionary perspective and for which there is actually some empirical evidence (e.g., Pingle and Mitchell, 2002; Johansson-Stenman and Martinsson, 2006), it is for this reason optimal to have a negative marginal capital income tax rate, that is, subsidize savings. Note also that a negative marginal capital income tax rate has similarities with a progressive consumption tax, as discussed by Frank (1999, 2007, 2008).

Finally, and by analogy with the results derived earlier, it follows that the marginal capital income tax rate is zero for both ability-types if the average net degree of positionality is equally large when young and when old, and if leisure is weakly separable from the other goods in the sense that the utility function can be written as \( U_t = u_t (c_t, x_t, \Delta_t^{i, cc}, \Delta_t^{i, xx}, z_t) \).

5.2. Upward Social Comparisons. There are several possible ways to model upward comparisons, for instance, that only the low-ability type makes such comparisons or that all individuals compare themselves with the average consumption among high-ability agents.\(^{20}\) We will choose the latter approach primarily because it is more general than the former (the latter encompasses the former as a special case). When each individual solely compares his/her own consumption with the average consumption of the high-ability type, the utility function of ability-type \( i \) born in period \( t \) can be written as

\[
U_t^i = v_t (c_t^i, z_t^i, x_t^i, c_t^{i, 2} - \bar{c}_t^{2, i}) = v_t (c_t^i, z_t^i, x_t^i, \Delta_t^{i, cc}, \Delta_t^{i, xx}) = u_t (c_t^i, z_t^i, x_t^i, \bar{c}_t, \bar{c}_t^{2, i}),
\]

where \( \bar{c}_t^2 = [n_t^2 c_t^2 + n_{t-1}^2 c_{t-1}^2] / [n_t^2 + n_{t-1}^2] \) represents the average consumption among high-ability individuals at time \( t \). We can then define the degree of upward consumption positionality when young and when old, respectively, as

\[
\alpha_t^{i, cc} = \frac{v_t^l \Delta_t^{i, cc}}{v_t^l \Delta_t^{i, cc} + v_t^l \Delta_t^{i, xx}},
\]

\[
\alpha_t^{i, xx} = \frac{v_t^l \Delta_t^{i, xx}}{v_t^l \Delta_t^{i, xx} + v_t^l \Delta_t^{i, xx}},
\]

where \( \alpha_t^{i, cc} \) reflects the fraction of the overall utility increase from the last dollar spent by an individual of ability-type \( i \) in period \( t \) that is due to the increased relative consumption compared to high-ability individuals; \( \alpha_t^{i, xx} \) reflects the corresponding fraction when old instead of when

\(^{20}\) Micheletto (2008) also analyzes upward comparisons in an economy with positional preferences. Yet, his framework differs from ours in several fundamental ways; he uses a static model without capital income taxation and only allows the low-ability type to be positional.
young. We can then define the average degree of upward consumption positionality as

$$\bar{\alpha}_i^2 = \sum_i \alpha_i^{12, x} n_{t-1}^{i} N_i + \sum_i \alpha_i^{12, c} n_{t}^{i} N_i$$

where $N_i = n_t^1 + n_t^2 + n_{t-1}^1 + n_{t-1}^2$ as before.

Note that all positional externalities are, in this case, generated by the behavior of the high-ability type. As a consequence, the marginal labor income and capital income tax rates implemented for the low-ability type will correspond to the standard model (in which goods are completely nonpositional). Thus, using the notation introduced above, we have $T_i (w_t^1 l_t^1) = \sigma_i^1$ and $\Phi'_{r+1}(r_{t+1}) = \delta_i^1$, respectively. Then, if we use the short notations,

$$\Gamma_i^2 = \frac{\lambda_{t-1} \hat{u}_{t-1,x}^2}{\gamma_t N_i} [\hat{\alpha}_i^{22,x} - \alpha_i^{12,x}] + \frac{\lambda_t \hat{u}_{t,c}^2}{\gamma_t N_i} [\hat{\alpha}_i^{22,c} - \alpha_i^{12,c}],$$

$$\omega_i = \frac{(1 - \bar{\alpha}_i^{22}) + (1/\kappa_i^2)(\bar{\alpha}_i^{2} - \Gamma_i^2)}{(1 - \bar{\alpha}_i^{22})},$$

where $\bar{\alpha}_i^{22} = (n_t^2 \alpha_i^{22,c} + n_{t-1}^2 \alpha_i^{22,x})/N_i$, $N_i^2 = n_t^2 + n_{t-1}^2$, and $\kappa_i^2 = N_i^2 / N_i$ is the share of high-ability agents, the corresponding tax formulas for the high-ability type can be written as

$$T_i (w_t^2 l_t^2) = \frac{1}{\omega_i \kappa_i^2} \left[ \alpha_i^2 - \frac{\Gamma_i^2}{1 - \bar{\alpha}_i^2} \right],$$

$$\Phi'_{r+1}(r_{t+1}) = \frac{1}{\omega_{r+1}} \left[ \frac{1}{r_{t+1}} \right] \left[ \frac{\bar{\alpha}_i^{22} - \Gamma_i^{2}}{1 - \bar{\alpha}_i^{22}} + \frac{1}{\kappa_i^{r+1}} \right] - \frac{\bar{\alpha}_i^2 - \Gamma_i^2}{1 - \bar{\alpha}_i^2} \frac{1}{\kappa_i^2}.$$  

Equations (34) and (35) closely resemble their counterparts in Section 4, and each formula permits an analogous interpretation in terms of the net degree of consumption positionality. However, there is one important difference: In this case, the optimal policy response to positional concerns is to modify the marginal income tax rates for the high-ability type only, whereas the corresponding marginal income tax rates for the low-ability type remain as in the standard model. The reason is, of course, that the low-ability type does not generate any positional externalities in this case.

The “positional arms race” per se may seem to be particularly detrimental for the low-ability type in the case of upward social comparisons, since the low-ability type is directly affected by the high-ability type’s consumption, whereas the high-ability type is not directly affected by the low-ability type’s consumption. Yet, the results here suggest that the potential redistributional effect (favoring the low-ability type) of an optimal income tax system that takes into account such positional externalities might be especially large in the case of upward comparisons, since the externality-correcting tax component will be paid only by the high-ability type.\(^{21}\)

6. CONCLUSION

As far as we know, this article is the first to consider optimal nonlinear income taxation in a second-best economy with asymmetric information, where people care about relative consumption, based on a dynamic (OLG) model. The model used is an extension of the standard optimal nonlinear income tax model with two ability types. Our article also recognizes the idea that each

\(^{21}\) Frank (2007, 2008) argues that the U.S. middle class has suffered severely from the positional arms race. In order to analyze more than two classes, we clearly need a richer model. However, it appears to us that several of the implications to the middle class discussed by Frank can be interpreted in terms of the low-ability type in our model.
individual may compare himself/herself more with some people than with others. Therefore, in addition to measures of reference consumption based on the average consumption in the economy as a whole, we also considered within-generation and upward comparisons.

We began by analyzing the case where the reference consumption is equal to the average consumption. Our results show that the more positional people are on average, ceteris paribus, the higher the marginal labor income tax rates. The intuition is that a higher marginal labor income tax rate reduces hours of work and, therefore, the resources available for private consumption. As a consequence, it also reduces the reference consumption to which people compare their own consumption. The effect of positional preferences on the marginal labor income tax rates also depends on whether the (mimicked) low-ability type is more or less positional than the mimicker, as this will determine whether an increase or a decrease in the reference consumption works to relax the self-selection constraint. Therefore, as the mimicker uses more leisure than the low-ability type (while mimicking the low-ability type in income-consumption space), our results suggest that the relationship between, on the one hand, the degree of positionality and, on the other, the use of leisure would constitute valuable information from the perspective of tax policy. By using the (scarce) available empirical evidence, our model implies that the optimal marginal labor income tax rates are likely to be much higher than suggested by models without relative consumption comparisons.

The effects of positional preferences on the marginal capital income tax rates are ambiguous in general. This also accords well with intuition, as the marginal capital income tax rates reflect a trade-off between present and future consumption. The more the average net degree of positionality (i.e., the average degree of positionality adjusted for the self-selection effect) increases (decreases) over time, ceteris paribus, the higher (lower) will be the marginal capital income tax rate. In the special case where the degree of positionality is constant over time and across agent types, plausible empirical estimates suggest that the marginal capital income tax rate of the low-ability type may be substantially smaller in absolute value than in the conventional optimal income tax model. In addition, if the degree of positionality is constant over time and across agent types, we are able to generalize the well-known result of Ordover and Phelps (1979) to a model with positional preferences; in other words, if leisure is weakly separable from the other goods in the utility function, then the marginal capital income tax rates should be zero for both ability types.

The alternative measures of reference consumption, that is, those based on within-generation or upward comparisons, allow us to explore some implications of the possibility that individuals compare themselves more with some people than with others. Within-generation comparisons give tax formulas in the same format as those based on average consumption comparisons. However, an important difference is that for within-generation comparisons, individuals only impose positional externalities on other people of the same generation; as a consequence, the policy rules for marginal income taxation ought to be interpreted in light of this more narrow interaction. For example, here it becomes important for marginal capital income taxation whether people become more or less positional as they grow old. If they become less positional, then, ceteris paribus, this is a case for negative marginal capital income taxes and vice versa.

In the case of upward comparisons, the marginal labor income and capital income tax rates of the low-ability type remain as in the standard model, whereas the corresponding marginal income tax rates for the high-ability type are modified, since each high-ability individual imposes positional externalities on other people. The intuition is that the behavior of the low-ability type does not contribute to the externalities generated in the positional arms race. As a consequence, internalization of positional externalities through income taxes has potentially very large redistributional effects in this case.

Finally, although this article in several respects generalizes the literature on optimal taxation when relative consumption matters, there are still many important aspects left to explore. Examples include a multi-country setting, public good provision in a dynamic economy, and the case where also relative leisure matters. We hope to address these issues in future research.
APPENDIX

A.1. Results of the Base Case; Sections 3 and 4.

A.1.1. First-order conditions. The first-order conditions for \( t_i^1, c_i^1, x_i^{1,1}, t_i^2, c_i^2, x_i^{1,1}, \) and \( K_{t+1} \) are given by

\[
\begin{align*}
(A.1) & \quad - \frac{\partial W}{\partial (n_i^1 U_i^1)} n_i^1 u_{i,z}^1 + \phi \lambda_i \hat{u}_{i,z} + \gamma_i n_i^1 w_i^1 = 0, \\
(A.2) & \quad \frac{\partial W}{\partial (n_i^1 U_i^1)} n_i^1 u_{i,c}^1 - \lambda_i \hat{u}_{i,c}^1 - \gamma_i n_i^1 + \frac{n_i^1}{N_i} \frac{\partial \mathcal{L}}{\partial \bar{c}_i} = 0, \\
(A.3) & \quad \frac{\partial W}{\partial (n_i^2 U_i^2)} n_i^1 u_{i,x}^1 - \lambda_i \hat{u}_{i,x}^1 - \gamma_i n_i^1 + \frac{n_i^1}{N_i} \frac{\partial \mathcal{L}}{\partial \bar{c}_i} = 0, \\
(A.4) & \quad \left[ \frac{\partial W}{\partial (n_i^1 U_i^1)} n_i^2 \right] u_{i,z}^2 + \gamma_i n_i^2 w_i^2 = 0, \\
(A.5) & \quad \left[ \frac{\partial W}{\partial (n_i^2 U_i^2)} n_i^2 \right] u_{i,c}^2 - \gamma_i n_i^2 + \frac{n_i^2}{N_i} \frac{\partial \mathcal{L}}{\partial \bar{c}_i} = 0, \\
(A.6) & \quad \left[ \frac{\partial W}{\partial (n_i^2 U_i^2)} n_i^2 \right] u_{i,x}^2 - \gamma_i n_i^2 + \frac{n_i^2}{N_i} \frac{\partial \mathcal{L}}{\partial \bar{c}_i} = 0, \\
(A.7) & \quad \gamma_{t+1} [1 + r_i] - \gamma_t = 0,
\end{align*}
\]

where we have used \( w_i^1 = F_i(L_i^1, L_i^2, K_i, t) \) for \( i = 1,2, \) and \( r_i = F_K(L_i^1, L_i^2, K_i, t) \) from the first-order conditions of the firm.

**Proof of Lemma 1.** From Equation (1) we have that \( u_{t,c}^i = v_{t,c}^i + v_{t,\Delta_t}^i, u_{t,x}^i = -v_{t,\Delta_t}^i, u_{t,x}^i = v_{t,x}^i + v_{t,\Delta_{t-1}}^i, \) and \( u_{t,\Delta_t}^i = -v_{t,\Delta_{t-1}}^i, \) so

\[
\begin{align*}
(A.8) & \quad u_{t,c}^i = -\alpha_{t,c}^i u_{t,c}^i, \\
(A.9) & \quad u_{t,x}^i = -\alpha_{t,x}^i u_{t,x}^i.
\end{align*}
\]

Corresponding expressions hold for the mimicker. By combining Equations (15), (A.8), and (A.9), and the corresponding expressions for the mimicker, we obtain

\[
\begin{align*}
(A.10) & \quad \frac{\partial \mathcal{L}}{\partial \bar{c}_i} = -\sum_{i=1} \frac{\partial W}{\partial n_i^1 u_{i-1,x}^1} n_i^1 \alpha_{i-1,x}^1 u_{i-1,x}^i - \sum_{i=1} \frac{\partial W}{\partial n_i^1 u_{i,c}^1} n_i^1 \alpha_{i,c}^1 u_{i,c}^i \\
& \quad - \lambda_{t-1} \left[ \alpha_{t}^{2,x} u_{t-1,x}^2 - \hat{\alpha}_{t}^{2,x} \bar{u}_{t-1,x}^2 \right] - \lambda_t \left[ \alpha_{t}^{2,c} u_{t,c}^2 - \hat{\alpha}_{t}^{2,c} \bar{u}_{t,c}^2 \right].
\end{align*}
\]

Note that Equations (A.2), (A.3), (A.5), and (A.6) imply

\[
\begin{align*}
(A.11) & \quad \frac{\partial W}{\partial n_i^1 U_i^1} n_i^1 u_{i,c}^1 = \lambda_i \hat{u}_{i,c} + \gamma_i n_i^1 - \frac{n_i^1}{N_i} \frac{\partial \mathcal{L}}{\partial \bar{c}_i}, \\
(A.12) & \quad \frac{\partial W}{\partial n_i^2 U_i^2} n_i^2 u_{i,c}^2 = -\lambda_i \hat{u}_{i,c} + \gamma_i n_i^2 - \frac{n_i^2}{N_i} \frac{\partial \mathcal{L}}{\partial \bar{c}_i}.
\end{align*}
\]
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Obtain be derived in a similar way. Equation (17). The marginal labor income tax rate of the high-ability type, Equation (18), can be derived in a similar way. By combining Equations (A.1) and (A.2), we obtain

(A.13) \[
\frac{\partial W}{\partial (n_{t-1}^1 U_{i-1}^1)} n_{i-1}^1 u_{i-1,x} = \lambda_{i-1} \hat{u}_{i-1,x} + \gamma_i n_{i-1}^1 - \frac{n_{i-1}^1}{N_i} \frac{\partial L}{\partial \hat{c}_i},
\]

(A.14) \[
\frac{\partial W}{\partial (n_{t-1}^2 U_{i-1}^2)} n_{i-1}^2 u_{i-1,x} = -\lambda_{i-1} \hat{u}_{i-1,x} + \gamma_i n_{i-1}^2 - \frac{n_{i-1}^2}{N_i} \frac{\partial L}{\partial \hat{c}_i}.
\]

Substituting Equations (A.11)–(A.14) and the definition of \( \Gamma_i \) into Equation (A.10) gives Equation (16). \( \blacksquare \)

A.1.2. The marginal labor income tax rates. Consider the tax formula for the low-ability type. By combining Equations (A.1) and (A.2), we obtain

(A.15) \[
\frac{u_{i,z}^1}{u_{i,c}^1} \left[ \lambda_i \hat{u}_{i,c} + \gamma_i n_{i}^1 - \frac{n_{i}^1}{N_i} \frac{\partial L}{\partial \hat{c}_i} \right] = \lambda_i \phi \hat{u}_{i,z}^1 + \gamma_i n_{i}^1 w_{i}^1.
\]

By substituting \( T_i(w_{i}^1 l_{i}^1) w_{i}^1 = w_{i}^1 - u_{i,z}^1 / u_{i,c}^1 \) into Equation (A.15) and rearranging, we obtain Equation (17). The marginal labor income tax rate of the high-ability type, Equation (18), can be derived in a similar way.

To derive Equation (19), we combine Equations (16) and (17) to obtain

(A.16) \[
T_i(w_{i}^1 l_{i}^1) = \frac{\lambda_i^*}{w_{i}^1 n_{i}^1} \left[ MRS_{c,c}^{1,1} - \phi MRS_{c,c}^{2,1} \right] - \frac{MRS_{c,c}^{1,1}}{\gamma_i w_{i}^1 N_i} \frac{1}{1 - \alpha_i} \left[ -\lambda_{i-1}^1 \left[ \alpha_i^{1,x} - \hat{\alpha}_i^{2,x} \right] \hat{u}_{i-1,x}^{2} - \lambda_i \left[ \alpha_i^{1,c} - \hat{\alpha}_i^{2,c} \right] \hat{u}_{i,c}^{2} - \gamma_i N_i \hat{\alpha}_i \right].
\]

Then, by using \( MRS_{c,c}^{1,1} / w_{i}^1 = 1 - T_i(w_{i}^1 l_{i}^1) \) and rearranging, we obtain Equation (19) for the low-ability type. The marginal labor income tax rate for the high-ability type can be derived in a similar way.

A.1.3. The marginal capital income tax rates. Let us consider the marginal capital income tax rate of the low-ability type. By combining Equations (A.2) and (A.3), we obtain

(A.17) \[
MRS_{c,x}^{1,1} \left[ \lambda_i \hat{u}_{i,c}^{2} + \gamma_i n_{i+1}^1 - \frac{n_{i+1}^1}{N_i+1} \frac{\partial L}{\partial \hat{c}_i} \right] = \lambda_i \hat{u}_{i,c}^{2} + \gamma_i n_{i+1}^1 - \frac{n_{i+1}^1}{N_i} \frac{\partial L}{\partial \hat{c}_i}.
\]

We then use Equations (7) and (A.7) to derive \( MRS_{c,x}^{1,1} = 1 + r_{i+1} - r_{i+1} \Phi_{i+1}^i (s_i r_{i+1}) \) and \( \gamma_i = \gamma_i + [1 + r_{i+1}] \), respectively. Substituting into Equation (A.17) and rearranging, we obtain Equation (20). Equation (21) can be derived in a similar way.

Substituting Equation (16), for period \( t \) and period \( t + 1 \), into Equations (20) and (21), we obtain

(A.18) \[
\Phi_{i+1}^i (s_i r_{i+1}) = \delta_i^i + \frac{\gamma_i}{\gamma_i + 1} \frac{\Gamma_i - \hat{\alpha}_i}{r_{i+1} + 1 - \hat{\alpha}_i} - \frac{1 + r_{i+1} \Gamma_{i+1} - \hat{\alpha}_{i+1}}{r_{i+1} + 1 - \hat{\alpha}_{i+1}} + \Phi_{i+1}^i (s_i r_{i+1}) \frac{\Gamma_{i+1} - \hat{\alpha}_{i+1}}{1 - \hat{\alpha}_{i+1}}
\]

for \( i = 1, 2 \), where we have used the short notations \( \delta_i^i \) and \( \delta_i^i \) as defined earlier. Now, using \( MRS_{c,x}^{1,1} = 1 + r_{i+1} - r_{i+1} \Phi_{i+1}^i (s_i r_{i+1}) \) together with \( (1 + r_{i+1}) = \gamma_i / \gamma_i + 1 \) in Equation (A.18) and rearranging, we obtain Equation (22).
The proof of Corollary 1 follows from acknowledging that the mimicker and the low-ability type differ only with respect to preferences and the use of leisure. Given the separability assumption and that consumers share a common subutility function \( f_i(\cdot) \), it follows that \( MR_{c,t}^{1,i} = MR_{c,t}^{2,i} \), which implies Corollary 1.

A.2. Results of the Case with Comparisons Within-Generations; Subsection 5.1.

A.2.1. First-order conditions and positionality effects. The first-order conditions for \( l_i^1, l_i^2 \), and \( K_{t+1} \) are the same as before, whereas the optimality conditions for \( c_i^1, x_{t+1}^1, c_i^2 \) and \( x_{t+1}^2 \) are given by

(A.23) \[ \frac{\partial L}{\partial \bar{c}_{t,t}} = \sum_{i=1}^{2} \frac{\partial W}{\partial (n_i^1 U_i^1)} n_i^1 u_i^1,_{c,t} - \lambda_i \dot{u}_{t,c}^2 - \gamma_i n_i^1 \frac{\partial L}{\partial \bar{c}_{t,t}} = 0, \]

(A.24) \[ \frac{\partial L}{\partial \dot{x}_{t,t-1}} = \sum_{i=1}^{2} \frac{\partial W}{\partial (n_i^1 U_i^1)} n_i^1 u_i^1,_{\dot{x}_{t,t}} + \lambda_i \dot{u}_{t,\dot{x}}^2_{t,t} + \lambda_{t-1} \left[ u_{t-1,\dot{x}_{t-1}}^2 - \dot{u}_{t-1,\dot{x}_{t-1}}^2 \right]. \]

From Equation (25) we have that \( u_{t,c}^i = v_{t,c}^i + u_{t,\Delta t}^i \) and \( u_{t,\dot{x}}^i = -v_{t,\Delta t}^i \), which together with Equation (26a) implies

(A.25) \[ u_{t,\dot{x}}^i = -\alpha_{t,cc}^i u_{t,c}^i, \]

whereas \( u_{t,\dot{x}_{t+1}}^i = -v_{t,\Delta t_{t+1}}^i \) together with Equation (26b) implies

(A.26) \[ u_{t,\dot{x}_{t+1}}^i = -\alpha_{t+1,xx}^i u_{t,x}^i. \]

Substituting Equations (A.25) and (A.26) into Equation (A.23) and (A.24), we obtain

(A.27) \[ \frac{\partial L}{\partial \bar{c}_{t,t}} = -\sum_{i=1}^{2} \frac{\partial W}{\partial (n_i^1 U_i^1)} n_i^1 x_{t,c}^i + \lambda_i \left[ \alpha_{t,cc}^i u_{t,c}^2 - \alpha_{t,cc}^i \dot{u}_{t,c}^2 \right]. \]
Rewriting Equations (A.19)–(A.22) in the same way as Equations (A.2), (A.3), (A.5), and (A.6) were rewritten into Equations (A.11)–(A.14), and substituting them into (A.27) and (A.28), implies Equations (27a) and (27b).

A.2.2. The marginal labor and capital income tax rates. The optimal marginal labor income tax rates for the low- and high-ability types are obtained by combining Equation (6) with (A.1) and (A.19), and with (A.4) and (A.21), respectively,

(A.29) \[ T_t^l(w_t^l l_t^l) = \frac{\lambda_t}{w_t^l n_t^l} \left[ MRS_{c,e}^{1,t} - \phi M\hat{R}_{2,c}^{2,t} \right] \frac{MRS_{c,e}^{1,t}}{\gamma_t w_t^l n_t^l} \frac{\partial L}{\partial \bar{c}_{t,i}}. \]

(A.30) \[ T_t^l(w_t^l l_t^l) = -\frac{MRS_{c,e}^{2,t}}{\gamma_t w_t^l n_t^l} \frac{\partial L}{\partial \bar{c}_{t,i}}. \]

Substituting Equation (27a) into Equations (A.29) and (A.30), using that \( MRS_{c,e}^{2,t}/w_t^l = 1 - T_t^l(w_t^l l_t^l) \) and rearranging then implies Equation (28).

By combining Equations (A.19) and (A.20), we obtain

(A.31) \[ MRS_{c,e}^{1,t} \left[ \lambda_t \hat{u}_{t,x}^2 + \gamma_{t+1} n_t^1 - \frac{n_t^1}{n_{t+1}} \frac{\partial L}{\partial \bar{c}_{t+1,i}} \right] = \lambda_t \hat{u}_{t,c}^2 + \gamma_t n_t^1 - \frac{n_t^1}{n_t} \frac{\partial L}{\partial \bar{c}_{t,i}}. \]

Substituting \( MRS_{c,e}^{1,t} = 1 + r_{t+1} - r_{t+1} \Phi_{t+1}^{s_t r_{t+1}} \) and \( \gamma_t = \gamma_{t+1}[1 + r_{t+1}] \), respectively, into Equation (A.31) and rearranging, we obtain Equation (29). Equation (30) is derived correspondingly for the high-ability type. Substituting Equations (27a) and (27b) into Equations (29) and (30), and using that \( MRS_{c,e}^{2,t} = 1 + r_{t+1} - r_{t+1} \Phi_{t+1}^{s_t r_{t+1}} \) together with \( 1 + r_{t+1} = \gamma_t/\gamma_{t+1} \), we obtain Equation (31).

A.3. Results of the Case with Upward Comparisons; Subsection 5.2.

A.3.1. First-order conditions and the positionality effect. The first-order conditions for \( l_t^l \), \( l_t^2 \), and \( K_{t+1} \) are again the same as before, whereas the optimality conditions for \( c_t^1, x_t^1, c_t^2, \) and \( x_{t+1}^2 \) are given by

(A.32) \[ \frac{\partial W}{\partial (n_t^1 U_t^1)} n_t^1 u_{t,c}^1 - \lambda_t \hat{u}_{t,c}^2 - \gamma_t n_t^1 = 0, \]

(A.33) \[ \frac{\partial W}{\partial (n_t^1 U_t^1)} n_t^1 u_{t,x}^1 - \lambda_t \hat{u}_{t,x}^2 - \gamma_t n_t^1 = 0, \]

(A.34) \[ \left[ \frac{\partial W}{\partial (n_t^2 U_t^2)} n_t^2 + \lambda_t \right] u_{t,c}^2 - \gamma_t n_t^2 + \frac{n_t^2}{N_t} \frac{\partial L}{\bar{c}_{t+1}^2} = 0, \]

(A.35) \[ \left[ \frac{\partial W}{\partial (n_t^2 U_t^2)} n_t^2 + \lambda_t \right] u_{t,x}^2 - \gamma_{t+1} n_t^2 + \frac{n_t^2}{N_t} \frac{\partial L}{\bar{c}_{t+1}^2} = 0, \]
where the welfare effect of a change in the average consumption among the high-ability individuals at time \( t \), \( c_{t, i}^2 \), can be written as

\[
\frac{\partial L}{\partial c_{t, i}^2} = \sum_{i=1}^{N_t} \frac{\partial W}{\partial (n_{t-1}^i U_{t-1}^i)} n_{t-1}^i u_{t-1, i}^i + \sum_{i=1}^{2} \frac{\partial W}{\partial (n_t^i U_t^i)} n_t^i u_{t, i}^i \\
+ \lambda_{t-1} \left[ u_{t-1, i}^2 - \tilde{u}_{t-1, i}^2 \right] + \lambda_t \left[ u_{t, i}^2 - \tilde{u}_{t, i}^2 \right].
\]

From Equation (32) we have \( u_{t, i}^i = v_{t, i}^i + v_{t, i, \Delta_{t, i}}^i, u_{t, i} = v_{t, i}^i + v_{t, i, \Delta_{t, i}}^i \), and \( u_{t, i, \Delta_{t, i}}^i = -v_{t, i, \Delta_{t, i}}^i \), which (with corresponding expressions for the mimicker) imply that \( u_{t, i}^i = -\alpha_{t, i}^{12, c} u_{t, c}^i \) and \( u_{t, i}^i = -\alpha_{t, i}^{12, x} u_{t, x}^i \). These expressions substituted into Equation (A.36) imply

\[
\frac{\partial L}{\partial c_{t, i}^2} = \sum_{i=1}^{2} \frac{\partial W}{\partial (n_{t-1}^i U_{t-1}^i)} n_{t-1}^i \alpha_{t, i}^{12, x} u_{t-1, x}^i - \sum_{i=1}^{2} \frac{\partial W}{\partial (n_t^i U_t^i)} n_t^i \alpha_{t, i}^{12, c} u_{t, c}^i \\
- \lambda_{t-1} \left[ \alpha_{t, i}^{22, x} u_{t-1, x}^i - \tilde{\alpha}_{t, i}^{22, x} \tilde{u}_{t-1, x}^i \right] - \lambda_t \left[ \alpha_{t, i}^{22, c} u_{t, c}^i - \tilde{\alpha}_{t, i}^{22, c} \tilde{u}_{t, c}^i \right].
\]

Rewriting Equations (A.32)–(A.35) in the same way as Equations (A.2), (A.3), (A.5), and (A.6) were rewritten into Equations (A.11)–(A.14), and substituting them into (A.37), implies

\[
\frac{\partial L}{\partial c_{t, i}^2} = -\frac{\alpha_{t, i}^{12, x} \gamma_t N_t}{1 - \alpha_{t, i}^{22, x}} + \frac{1}{1 - \alpha_{t, i}^{22, x}} \left[ \lambda_{t-1} \tilde{u}_{t-1, x}^i \left( \tilde{\alpha}_{t, i}^{22, x} - \alpha_{t, i}^{12, x} \right) + \lambda_t \tilde{u}_{t, c}^i \left( \alpha_{t, i}^{22, c} - \tilde{\alpha}_{t, i}^{22, c} \right) \right].
\]

A3.2. The marginal labor and capital income tax rates. The optimal marginal labor income tax rates for the low- and high-ability types are obtained by combining Equation (6) with (A.1) and (A.32), and with (A.4) and (A.34), respectively,

\[
T_t^l (w_t^l l_t^l) = \frac{\lambda_t^n}{w_t^l n_t^l} \left[ MRS_{c, c}^{1, l} - \phi MRS_{c, c}^{2, l} \right],
\]

\[
T_t^l (w_t^l l_t^l) = -\frac{MRS_{c, c}^{2, l}}{\gamma_t w_t^l n_t^l} \frac{\partial L}{\partial c_{t, i}^2}.
\]

Substituting Equation (A.38) into Equation (A.40), using that \( MRS_{c, c}^{2, l} / w_t^l = 1 - T_t^l (w_t^l l_t^l) \) and rearranging, then implies Equation (34).

By combining Equations (A.32) and (A.33), and (A.34) and (A.35), respectively, we obtain

\[
MRS_{c, c}^{1, l} \left[ \lambda_t \tilde{u}_{t, x}^i + \gamma_t n_t^l \right] = \lambda_t \tilde{u}_{t, c}^i + \gamma_t n_t^l,
\]

\[
MRS_{c, c}^{2, l} \left[ \gamma_t n_t^l - \frac{n_t^l}{\gamma_t} \frac{\partial L}{\partial c_{t+1}^2} \right] = \gamma_t n_t^l - \frac{n_t^l}{\gamma_t} \frac{\partial L}{\partial c_{t+1}^2}.
\]

Using that \( MRS_{c, c}^{1, l} = 1 + r_{t+1} - r_{t+1} \Phi_{t+1}^l (s_{t+1}^l r_{t+1}) \) together with \( [1 + r_{t+1}] = \gamma_t / \gamma_{t+1} \) and substituting Equation (A.38) into Equation (A.42), we obtain

\[
\Phi_{t+1}^l (s_{t+1}^{l, r_{t+1}}) = \frac{\lambda_t \tilde{u}_{t, x}^i}{\gamma_t n_t^l r_{t+1}} \left[ MRS_{c, c}^{1, l} - MRS_{c, c}^{2, l} \right],
\]

and Equation (35).
REFERENCES


