Games and Economic Behavior 68 (2010) 208–219

Contents lists available at ScienceDirect

Games and Economic Behavior

www.elsevier.com/locate/geb

Risk aversion and expected utility of consumption over time

Olof Johansson-Stenman

Department of Economics, School of Business, Economics and Law, University of Gothenburg, Box 640, SE-40530 Gothenburg, Sweden

ARTICLE INFO

Article history:
Received 11 August 2006
Available online 22 July 2009

JEL classification:
D81
D91

Keywords:
Expected utility of income
Expected utility of final wealth
Dynamic consumption theory
Asset integration
Time inconsistency
Narrow bracketing

ABSTRACT

The calibration theorem by Rabin [Rabin, M., 2000a. Risk aversion and expected utility theory: A calibration theorem. Econometrica 68, 1281–1292; Rabin, M., 2000b. Diminishing marginal utility of wealth cannot explain risk aversion. In: Kahneman, D., Tversky, A. (Eds.), Choices, Values and Frames. Cambridge University Press] implies that seemingly plausible small-stake choices under risk imply implausible large-stake risk aversion. This theorem is derived based on the expected utility of *wealth* model. However, Cox and Sadiraj [Cox, J.C., Sadiraj, V., 2006. Small- and large-stakes risk aversion: Implications of concavity calibration for decision theory. Games Econ. Behav. 56, 45–60] show that such implications do not follow from the expected utility of *income* model. One may then wonder about the implications for more applied consumption analysis. The present paper therefore expresses utility as a function of *consumption* in a standard life cycle model, and illustrates the implications of this model with experimental small- and intermediate-stake risk data from Holt and Laury [Holt, C.A., Laury, S.K., 2002. Risk aversion and incentive effects. Amer. Econ. Rev. 92, 1644–1655]. The results suggest implausible risk aversion parameters as well as unreasonable implications for long-term risky choices. Thus, the conventional intertemporal consumption model under risk appears to be inconsistent with the data.

1. Introduction

How well expected utility (EU) theory describes human behavior in general, including in small- and intermediate-stake gambles, has recently been discussed intensively. At the core is what expected utility is expressed as a function of. This paper provides a simple extension of some important aspects of this discussion to a life cycle setting where people derive utility from consumption (instead of wealth or payoffs), and illustrates this with numerical implications based on experimental data from Holt and Laury (2002).

Rabin (2000a) presents an important theoretical contribution in terms of a calibration theorem that implies conclusions of the following kind: *If for all wealth levels an expected utility maximizing person turns down a 50–50 lose $100/gain $200 gamble, he would also turn down a 50–50 lose $200/gain $20,000 gamble*. While it may seem plausible that some people would turn down the first gamble (for all wealth levels), it seems much less reasonable to turn down the second. According to Rabin and Thaler (2001, p. 206): "Even a *lousy* lawyer could have you declared legally insane for turning down *this* bet."[1] An important

E-mail address: Olof.Johansson@economics.gu.se.

[1] Given that "expected utility" refers to "expected utility of *wealth*," it is actually straightforward to derive an even stronger conclusion, as follows: "If for all wealth levels an expected utility maximizing person turns down a 50–50 lose $100/gain $200 gamble, he would also turn down a 50–50 lose $200/gain infinity gamble." Let K denote the gain in utility U from a wealth increase from w to $w+200$, where w is initial wealth. Then if the individual turns down a 50–50 lose $100/gain $200 gamble, it follows by concavity that the utility loss from a wealth change from w to $w-200$ is at least $2K$. Since this holds for all initial wealth levels it would also hold for the initial wealth $w+200$; hence we know that a wealth increase from $w+200$ to $w+400$ implies a U increase of less then $K/2$, and that a wealth increase from $w+200(r-1)$ to $w+200r$, where r is an arbitrary positive integer larger than 1, implies a utility increase of less than $(K/2)^{r-1}$. Hence, the utility change for a wealth increase from w to $w+200r$ is less than $\sum_{i=0}^{r-1} K/2^i = 2(1-0.5^r)K$.

0899-8256/$ – see front matter
doi:10.1016/j.geb.2009.07.001

feature of this calibration theorem is that it does not assume anything regarding the functional form of the utility function. However, the "for all wealth levels" part of the theorem is important. Although one can derive less extreme versions without this assumption, one must still assume that the individual would have made the same choice had he/she been substantially wealthier than what he/she actually is (see Rabin and Thaler, 2001 and Footnote 1 in the present paper). Largely based on the implications of this theorem, Rabin (2000a, 2000b) and Rabin and Thaler (2001) argue more generally that EU theory cannot explain behavior based on small-stake gambles, and hence that we need some other theory; they suggest a combination of loss aversion and mental accounting.

However, Cox and Sadiraj (2006) question this conclusion in a recent paper. They show that for the small-stake risk aversion assumption of Rabin (2000a), implausible large-stake risk aversion would *not* follow for the expected utility of *income* (EUI) model, where utility is expressed as a function of payoffs, in contrast to the expected utility of *final wealth* (EUW) model.[2] Moreover, since the global small-stake risk aversion assumed by Rabin (2000a) has no implication for the EUI model, it has no general implication for EU theory either. It is clear that Cox and Sadiraj have a valid and important point since EU theory is very general and builds on a set of axioms that do not preclude that utility may depend on wealth, income, experimental payoffs, or almost any state variable.[3]

In light of the findings by Cox and Sadiraj, one may be inclined to conclude that what has become known as *the Rabin critique* is overstated. Perhaps applied economists interested in measuring people's risk preferences or analyzing behavior based on existing estimates can ignore the Rabin critique and continue to interpret their results in terms of the concavity of universally valid utility functions. However, the results in this paper suggest that such a conclusion would be premature.

In applied economic analysis, people often make decisions over time, deriving instantaneous utility based on their present consumption level. Under risk, the conventional assumption is then that people maximize the expected present value of future instantaneous utility (e.g., Deaton, 1992; Gollier, 2001). We will denote this model the expected utility of consumption over time (EUCT) model, and take it as our point of departure. An obvious example where both time and risk are essential is how to best invest retirement savings; see, e.g., Gomes and Michaelides (2005).[4] In the EUCT model, utility is expressed as a function of a flow variable (unlike the EUW model), i.e., consumption, and implies complete asset integration (as in the EUW model), meaning that gains from a risky choice are treated exactly the same as income or wealth obtained in any other way.

The main contributions of the present paper can be summarized as follows: The respective relations between the EUCT model and the EUW and EUI models are analyzed in Section 2. It is concluded that the EUCT model is essentially equivalent to the EUW model when the wealth measure in the EUW model consists of the present value of all future consumption or income. In addition, it is shown that the functional form of the instantaneous utility function (expressed as a function of current consumption) carries over in a straightforward way to the utility function (expressed as a function of the present value of all future incomes) for instantaneous utility functions characterized by hyperbolic absolute risk aversion (HARA), which is a flexible functional form that includes constant relative risk aversion (CRRA) and constant absolute risk aversion (CARA) functions as special cases (Merton, 1971).

Then, by using data from a careful experimental study by Holt and Laury (2002), Section 3 analyzes whether observed behavior in small- and intermediate-stake size risk experiments can be reconciled by the EUCT model. The answer is no. The calculated implicit risk aversion parameters are found to be unreasonably large, and therefore cannot constitute concavity measures of universally valid instantaneous utility functions. Moreover, strong implications are derived with respect to what these degrees of risk aversion would correspond to for long-term risky choices in terms of future income or consumption levels. For example, whether based on CRRA or CARA preferences, a majority of the subjects from Holt and Laury would (given that they are EUCT maximizers) in the base case prefer an income level that with certainty would enable them to for the rest of their lifetime consume 36,000 USD annually, rather than a risky alternative where they with a 1% probability would be able to consume 35,990 USD annually and with a 99% probability an infinite amount. This clearly seems implausible. Similar implausible results are also obtained for the broader class of HARA preferences.

Compared to the results based on the EUW model by Rabin (2000a, 2000b) and Rabin and Thaler (2001), the results in the present paper are less general in the sense that they depend on specific functional forms. On the other hand, the results are less restrictive in the sense that they do not rely on any assumption that the choices would have been the same for all lifetime wealth levels, or for any higher lifetime wealth levels than the individuals currently have or expect

---

Consequently, the expected utility change of a 50–50 lose \$200/gain \$200r gamble is less than $(1-0.5^r)K - K = -0.5^r K$. Thus, the expected utility change is negative irrespective of r, i.e., irrespective of the gain. (One can easily obtain less extreme versions by replacing the "for all wealth levels" with "for wealth levels up to $w + \Delta w$.") Moreover, by replacing \$100 with an arbitrary positive number A in the above analysis, it follows more generally that, "If for all wealth levels an expected utility maximizing person turns down a 50–50 lose A/gain $2A$ gamble, he would also turn down a 50–50 lose $2A$/gain infinity gamble."

[2] Although it is clear from Rabin (2000a, 2000b) and Rabin and Thaler (2001, 2002) that they focus on the EUW model in their analyses, some of their statements have been interpreted to imply a criticism of expected utility theory more generally. Cox and Sadiraj (2006) also consider a more general two-argument model where utility depends on both initial wealth and payoff.

[3] Samuelson (2005), Rubinstein (2006), and Harrison et al. (2007) provide similar arguments.

[4] Long-run environmental problems, such as the greenhouse effect, constitute another important example where both time and risk are crucial. It is also typically shown that the profitability of extensive abatement today depends critically on the discount rate chosen, which in turn depends strongly on the concavity of the instantaneous utility function; see, e.g., Stern (2007) and Nordhaus (2007).

to obtain.[5] Section 4 concludes that the standard EUCT model appears inconsistent with available experimental small- and intermediate-stake data.

2. The EUCT model

The standard approach when dealing with intertemporal choices under risk is to maximize the expected present value of future instantaneous utility (e.g., Deaton, 1992; Gollier, 2001). Let us start with the intertemporal consumption choice under certainty and in the next step take risky decisions into account.

2.1. The intertemporal choice problem and HARA preferences

Consider an individual who experiences the instantaneous utility $u(c_t)$ at time t (from now), where u is increasing and strictly concave, and who will live for T more years. Assume further an additive and time-consistent utility formulation, such that the individual will maximize

$$U = \int_0^T u(c_t)e^{-\rho t}\,dt, \tag{1}$$

where ρ is the pure rate of time preference, sometimes denoted the utility discount rate. We will refer to U as utility. Under certainty, U is purely ordinal (in contrast to u), so that any monotonic transformation of U is permissible and hence constitutes an equally valid measure of utility.[6] The intertemporal budget constraint implies that the present value of future consumption equals the present value of future income, so that

$$\int_0^T c_t e^{-rt}\,dt = \int_0^T y_t e^{-rt}\,dt \equiv Y, \tag{2}$$

where r is the market interest rate. The associated Lagrangean can then be written as

$$\mathcal{L} = \int_0^T u(c_t)e^{-\rho t}\,dt + \lambda \int_0^T (y_t - c_t)e^{-rt}\,dt,$$

implying the corresponding first-order conditions

$$u'(c_t) = \lambda e^{(\rho - r)t} = u'(c_0)e^{(\rho - r)t}, \tag{3}$$

which together with the budget restriction determine the optimal consumption path.[7] Since the individual maximizes U given a certain present value of lifetime income Y, we can alternatively write $U = V(Y)$ for a fixed interest rate.

We will subsequently analyze what choices between small-stake lotteries imply in terms of risk aversion measures when people are EUCT maximizers, and also what they imply in terms of large-stake choices. In doing this, we would like to know under which conditions the functional form carries over from u to V. For example, if u is CRRA, can we then know that also V is CRRA? If this is the case (and it turns out that it is), it simplifies the analysis largely, since it is then straightforward to reduce the dynamic problem to a static analogue so that we in our numerical analysis can work directly with Y, or its corresponding annuity. Let c_t^* be the optimal consumption level at time t, and c^0 be the annuity corresponding to Y, meaning that an individual can afford exactly the constant consumption level c^0 for the rest of his/her lifetime, where hence $c^0 \equiv Y/S$ where $S \equiv (1 - e^{-rT})/r$ is the annuity factor. We can then prove (see Appendix A) the following result:

Proposition 1. *If u is HARA such that $u(c_t) = (\alpha + \beta c_t)^{(\beta-1)/\beta}/(\beta - 1)$, where $\alpha > -\beta c_t$, then*

$$i.\quad U = V(Y) = K\frac{(S\alpha + \beta Y)^{(\beta-1)/\beta}}{\beta - 1},$$

$$ii.\quad U = K'\frac{(\alpha + \beta c^0)^{(\beta-1)/\beta}}{\beta - 1} = K'u\big(c^0\big),$$

where K and K' are constants.

[5] However, note that the choices for CARA preferences would have been the same for all lifetime wealth levels. When utility is CRRA, by contrast, we know that an individual who is indifferent between accepting and not accepting a risky gamble would always accept it for wealth levels higher than his/her current level.

[6] This means for example that under certainty, $\breve{U} \equiv \ln U$ is just as valid of a measure of utility as U, in the sense that an individual who chooses a consumption path in order to maximize $\breve{U}$ will also maximize U. However, the only transformations of u that leave the optimal consumption path unaffected are affine transformations; hence u is cardinal and unique only up to affine transformations.

[7] For example, when $\rho = r$ it follows that $u'(c_t) = \lambda$, implying that also c_t is constant over time. Intuitively, people want to smooth their consumption over their life cycle in order to equalize their marginal instantaneous utility of income. This is also a standard result in dynamic consumption theory (e.g., Hall, 1978).

The *i*-part means that if u is HARA in c_t then V is indeed also HARA in Y, where moreover the only parameter change is that α is replaced by $S\alpha$. The *ii*-part means that when u is HARA we can measure utility (i.e., the present value of the instantaneous utility over the whole lifetime period) by using the instantaneous utility function, u, expressed as a function of the annuity, c^0; this will be used repeatedly in Section 3. Note that this holds regardless of whether a constant consumption path is optimal, i.e., regardless of whether $\rho = r$.

It is straightforward to verify that the HARA instantaneous utility function includes as special cases most functional forms that are typically used in applied analysis, including CRRA, CARA and the quadratic form (but not the so-called Expo-power form; cf. Saha, 1993). When u is HARA in c_t as described above, we obtain the Arrow–Pratt coefficient of absolute risk aversion as

$$A_t \equiv -\frac{u''(c_t)}{u'(c_t)} = \frac{1}{\alpha + \beta c_t},$$

whereas the corresponding coefficient of relative risk aversion is given by

$$R_t \equiv -\frac{u''(c_t)}{u'(c_t)} c_t = \frac{c_t}{\alpha + \beta c_t}.$$

It follows that u is characterized by CRRA for the special case when $\alpha = 0$, implying that

$$u(c_t) = \frac{c_t^{(\beta-1)/\beta}}{\beta - 1} = \frac{c_t^{1-R}}{1-R},$$

where $R = 1/\beta$. Similarly, u converges toward CARA when β approaches 0, so

$$u(c_t) = -\alpha e^{-c_t/\alpha} = -\frac{e^{-Ac_t}}{A},$$

where $A = 1/\alpha$. Using Proposition 1, we can then write utility in the CRRA case as

$$U = K' \frac{(c^0)^{1-R}}{1-R},$$

and in the CARA case as

$$U = -K' \frac{e^{-Ac^0}}{A}.$$

2.2. Introducing risk

Consider now a finite lottery with the income path y_t^i for $t \geqslant 0$ with probability p^i, where the realized income path is revealed before the consumption path is chosen. Expected utility is then given by

$$EU = \sum_{i=1}^{n} \left(\int_0^T u\left(c_t^{*i}\right) e^{-\rho t}\, dt \right) p^i = \sum_{i=1}^{n} V\left(Y^i\right) p^i, \tag{4}$$

where $Y^i \equiv \int_0^T y_t^i e^{-rt}\, dt$ is the present value of future incomes if state i occurs, and where each element of the optimal consumption path, c_t^{*i}, will satisfy (3). Note that under risk, each possible utility outcome, $U^i \equiv V(Y^i)$, must be interpreted in a cardinal sense, so that only affine transformations are permissible. Still, since the multiplications in Proposition 1 by K and K', respectively, are clearly affine transformations, the measures given in *i* and *ii* are therefore also valid as von Neumann–Morgenstern utility under risk.[8] Again, we see from (4) that the EUCT model is equivalent to the EUW model in the case where wealth is defined as the present value of all future incomes. Note that (4) holds generally, whereas in the case of HARA preferences we also have that the functional form carries over from u to V.

Assuming that the potential outcomes from the lottery (which can be positive or negative) occur today, we can write expected utility as $EU = \sum_{i=1}^{n} V(Y + x^i) p^i$, where x^i is the outcome of the lottery in state i and Y is the present value of all future income in the absence of the lottery. From Proposition 1 we have that when u is HARA as given above it follows that expected utility is given by

$$EU = \sum_{i=1}^{n} p^i \frac{(\alpha + \beta(c^0 + x^i/S))^{(\beta-1)/\beta}}{\beta - 1} \tag{5}$$

[8] Only affine transformations of U^i , describing the utility in state i, leave the choice unchanged, and are hence permissible; consequently U^i is cardinal. However, the optimal consumption choices are clearly unaffected by any monotonic transformation of EU; hence EU is ordinal.

(or by any monotonically increasing transformation of this expression). We can then write expected utility in the CRRA and CARA cases, respectively, as

$$EU=\sum_{i=1}^{n} p^{i}\frac{(c^{0}+x^{i}/S)^{1-R}}{1-R}, \tag{6}$$

$$EU=\sum_{i=1}^{n} p^{i}e^{-Ax^{i}/S}. \tag{7}$$

These expressions will be used in the numerical calculations in Section 3.

According to the so-called Arrow–Pratt approximation (see, e.g., Gollier, 2001, p. 22), we have generally that for small risks the risk premium ψ is approximately given by $\psi \approx A\frac{\mathrm{var}(x)}{2}$, so that $A \approx 2\frac{\psi}{\mathrm{var}(x)}$ and hence $R \approx 2\frac{\psi}{\mathrm{std}(x)}\frac{Sc^{0}}{\mathrm{std}(x)}$, where $A \equiv -V''/V'$ and $R \equiv -YV''/V'$ are the associated coefficients of absolute and relative risk aversion, respectively. The literature based on life cycle consumption behavior often refers to values of R in the 0.5–3 range.[9] According to Kocherlakota (1996, p. 52), "A vast majority of economists believe that values above 10 (or, for that matter, above 5) imply highly implausible behavior." The ratio between the present value of all future consumption and the standard deviation of the monetary outcome of a risk experiment is typically very large. This implies that unless the risk premium is a tiny fraction of the standard deviation of the monetary outcome, the behavior in the risk experiment will be described as "highly implausible" by the above quotation (which will be illustrated further in Section 3).

3. Numerical illustration based on data from Holt and Laury (2002)

There are many suitable experimental studies that could be used to illustrate the implications of the above model, but let us here rely on the well-known and carefully undertaken study by Holt and Laury (2002), who elicited the risk preferences of (mainly) US university students by using real money experiments with different stake sizes. Each student made a number of pairwise choices between one less risky (Option I) and one more risky (Option II) lottery; see Table 1 for a relevant subset. Indifference between Option I and Option II then implies a certain degree of risk aversion, and the choices were ordered so that indifference between the options implies stronger and stronger risk aversion. By observing at what point a subject switched to Option II, they obtained a risk aversion range in which he/she belonged. Holt and Laury used several different functional forms, including the flexible expo-power functional form that includes CRRA and CARA as special cases, but did not integrate the gains with other expected lifetime incomes, i.e., in line with the EUCT model.

3.1. CARA and CRRA preferences

In order to test the implications of the EUCT model with real data, let us first focus on the two most commonly used functional forms, CRRA and CARA,[10] where the instantaneous utility function can hence be written as $u_t=c_t^{1-R}/(1-R)$ and $u_t=-e^{-Ac_t}$, respectively. When an individual is indifferent between two lotteries, I and II, it then follows from (6) and (7):

$$\sum_{i=1}^{n} p_{\mathrm{I}}^{i}\left(c^{0}+x_{\mathrm{I}}^{i}/S\right)^{1-R}=\sum_{i=1}^{n} p_{\mathrm{II}}^{i}\left(c^{0}+x_{\mathrm{II}}^{i}/S\right)^{1-R}, \tag{8}$$

$$\sum_{i=1}^{n} p_{\mathrm{I}}^{i}e^{-Ax_{\mathrm{I}}^{i}/S}=\sum_{i=1}^{n} p_{\mathrm{II}}^{i}e^{-Ax_{\mathrm{II}}^{i}/S}. \tag{9}$$

From (8) and (9) we can easily solve numerically for R and A.

Consider now for comparison the EUI model where the lotteries are evaluated in isolation, and hence are independent of other incomes. The EUI model is therefore of course in general not consistent with EUCT. In the CRRA case we have:

$$\sum_{i=1}^{n} p_{\mathrm{I}}^{i}\left(x_{\mathrm{I}}^{i}\right)^{1-R}=\sum_{i=1}^{n} p_{\mathrm{II}}^{i}\left(x_{\mathrm{II}}^{i}\right)^{1-R}. \tag{10}$$

Clearly, since x is typically small compared to $Sc^{0} \equiv Y$, (10) should generally result in a smaller R than (8) when indifferent between the two lotteries. However, in the CARA case, where initial wealth does not affect choices, (9) still holds (corrected for the scale of A). The reason is of course that the expected utility change of a lottery is here independent of the initial

[9] For example, Blundell et al. (1994) and Attanasio and Browning (1995) found, in most of their estimates, R to be in the order of magnitude of 1 or slightly above. Vissing-Jørgensen (2002) found that R differs between stockholders (approx. 2.5 to 3) and bond holders (approx. 1 to 1.2).

[10] Following convention, these names simply reflect the functional form of the instantaneous utility function. What these functional forms imply in terms of actual choices under risk of course depends on other assumptions of the model as well.

Table 1
Calculated implicit parameters of absolute and relative risk aversion when people are indifferent between Options I and II, where the empirical results are taken from Holt and Laury (2002), for different cases.

Option I	Option II	Fraction choosing I	Implicit parameter of absolute risk aversion A if indifference between I and II	Implicit parameter of relative risk aversion R if indifference between I and II		
				EUI	EUCT base case	EUCT $r = 500\%$
Low-payoff lottery choices						
5/10 of 2 USD, 5/10 of 1.6 USD	5/10 of 3.85 USD, 5/10 of 0.1 USD	66%	0.101	0.146	19,248	202.8
6/10 of 2 USD, 4/10 of 1.6 USD	6/10 of 3.85 USD, 4/10 of 0.1 USD	40%	0.299	0.411	56,735	597.7
7/10 of 2 USD, 3/10 of 1.6 USD	7/10 of 3.85 USD, 3/10 of 0.1 USD	17%	0.516	0.676	97,980	1032.1
High-payoff lottery choices						
5/10 of 40 USD, 5/10 of 32 USD	5/10 of 77 USD, 5/10 of 2 USD	81%	0.005	0.146	962.8	23.15
6/10 of 40 USD, 4/10 of 32 USD	6/10 of 77 USD, 4/10 of 2 USD	62%	0.015	0.411	2838	45.24
7/10 of 40 USD, 3/10 of 32 USD	7/10 of 77 USD, 3/10 of 2 USD	39%	0.026	0.676	4900	69.75

Note: A is expressed in terms of the payoffs. All numerical calculations are performed in *Mathematica*.

wealth level, which is only true for CARA preference. Consequently, the EUI model is equivalent to the EUCT model for the CARA (and only the CARA) instantaneous utility function.

Consider first for comparison the result of the EUI model, where the experimental gains are evaluated independently of people's baseline income and wealth levels. It can be observed from Table 1 that based on the CARA preferences as expressed in (9), the median parameter of absolute risk aversion A is between 0.101 and 0.299 based on the low-stake lottery, and between 0.015 and 0.026 based on the high-stake lottery. Based on CRRA preferences, the median parameter of relative risk aversion R is calculated from (10) to be between 0.146 and 0.411 based on the low-stake lottery, and between 0.411 and 0.676 based on the high-stake lottery.

Consider now the conventional EUCT model. In the CARA case, A of course remains the same since with CARA preferences, choices between risky options are independent of initial wealth; cf., e.g., Rabin and Weizsäcker (forthcoming). Since the parameter estimates differ largely between the high- and low-stake lotteries, this suggests that CARA does not constitute a good approximation of subject preferences. However, the main concern here is whether the orders of magnitude constitute reasonable reflections of globally valid instantaneous utility functions. In the CRRA case, we clearly need estimates of S and c^0 in order to solve for R in (8). Let us therefore assume that the subjects are 20 years old, that they expect to live until they are 80 (i.e., that they have 60 years left), that the real market interest rate is 5% annually, and that they quite pessimistically will earn future incomes that will enable them to consume $c^0 = 10{,}000$ USD per year (at today's price level). For example, the second high-payoff lottery in the Holt and Laury experiment corresponds then to a lottery between the present values of future incomes, such that the subjects in option I can afford a constant annual consumption of 10,002.1048 USD with probability 0.6 and 10,001.6839 USD with probability 0.4, and in option II 10,004.0518 USD with probability 0.6 and 10,000.10524 USD with probability 0.4.

As can be seen in Table 1, the median R is now larger than 19,000 based on the low-stake lottery and larger than 2800 based on the high-stake lottery. These are clearly values way above what is generally considered to be plausible, i.e., values in the range of 0.5 to 3 or in any case considerably smaller than 10.[11] Note that we have made no assumption regarding the pure rate of time preference ρ, and all results are independent of whether the students actually would prefer to have a future increasing or decreasing consumption path over time. If the future annual consumption of the subjects were larger than 10,000 USD, then the implicit parameters of relative risk aversion would of course be even larger.

However, one may also believe that students have liquidity constraints and hence face a higher real interest rate than others. Let us therefore make the extreme assumptions of an annual real interest rate of 500% (instead of 5%). Solving for R in (8) nevertheless again reveals absurdly large values, as the last column of Table 1 shows.

Thus, we have seen that the choices in Holt and Laury imply absurdly large risk aversion coefficients if based on CRRA preferences, whereas the coefficients are identical between the EUI model and the EUCT model in the case of CARA preferences.

However, since A is not dimension free, it may be difficult to have a good intuition about what a reasonable range of A is. One perhaps tempting interpretation could be that the EUCT model works perfectly fine, but that people have CARA preferences (or similar) rather than CRRA preferences. However, even if one is willing to ignore the A discrepancies between the small- and large-stake experiments, this is not a plausible conclusion. To see this, consider the following gamble: In a safe alternative the individual would obtain the present value of all future income equal to 5 million USD. In a risky alternative, the individual would instead with a probability of 1% obtain 4.9999 million USD and with a 99% probability obtain an infinite amount. Presumably, most people would prefer the risky alternative. However, an individual with $A = 0.101$ would actually prefer the safe alternative.[12] Hence, $A = 0.101$ is indeed unreasonably large. Next, we will

[11] Independent of this study, Schechter (2007) also obtained absurdly large parameters of relative risk aversion in a risk experiment based on a sample in rural Paraguay.

[12] This is because $-e^{-0.101\cdot 5\cdot 10^6} > -0.01e^{-0.101\cdot 4.9999\cdot 10^6} - 0.99\cdot 0$.

Table 2
Calculated implicit annual rest-of-life consumption in the unlucky outcome based on the EUCT model, so that the degrees of risk aversion correspond to the ones obtained in Table 1.

Safe option USD/year	Risky option					Fraction that would choose the safe option
	Lucky outcome (probability = 99%) USD/year	Unlucky outcome (probability = 1%), USD/year				
		CARA		CRRA		
		$r = 5\%$	$r = 500\%$	$r = 5\%$	$r = 500\%$	
		Based on low-payoff lottery choices				
36,000	Infinite	35,999.0	35,901	35,991	35,188	66%
36,000	Infinite	35,999.6	35,967	35,997	35,723	40%
36,000	Infinite	35,999.8	35,981	35,998	35,840	17%
		Based on high-payoff lottery choices				
36,000	Infinite	35,979	34,000	35,828	29,242	81%
36,000	Infinite	35,993	35,333	35,942	32,441	62%
36,000	Infinite	35,996	35,615	35,966	33,668	39%

more systematically look into the implications for large-stake lottery choices expressed in terms of future consumption possibilities.

Consider the choice between a safe and a risky option concerning a subject's future income. In the safe option, he/she will with certainty for the rest of his/her life earn an amount corresponding to a constant annual consumption of c^S. In the risky option, he/she will with probability p obtain a high future income level that corresponds to a constant annual consumption level of c^H and with probability $1-p$ a low future income corresponding to the constant annual consumption level c^L. We can then solve for c^L from (8) and (9) for the CRRA and the CARA cases as follows:

$$c^L = \left(\frac{(c^S)^{1-R} - p(c^H)^{1-R}}{1-p}\right)^{\frac{1}{1-R}}, \tag{11}$$

$$c^L = \frac{1}{A}\ln\left(\frac{1-p}{e^{-Ac^S} - pe^{-Ac^H}}\right). \tag{12}$$

In the special case where the "lucky" outcome implies an infinite consumption level, and where $R > 1$ and $A > 0$, (11) and (12) reduce to:

$$c^L = (1-p)^{-1/(1-R)} c^S, \tag{13}$$

$$c^L = c^S + \frac{1}{A}\ln(1-p). \tag{14}$$

Table 2 illustrates the case where the lucky consumption level is infinite, and where moreover the probability of a lucky outcome is as high as 99%. Consider first the CRRA case with a 5% interest rate. The first line of Table 2 reveals that indifference between the safe and the risky option implies the same R as indifference between Option I and Option II in Table 1. Consequently, if people's behavior can be described by the EUCT model with CRRA preferences, then the same fraction (66%) would prefer the less risky option. This means that 66% of the subjects in Holt and Laury would actually prefer being able to consume 36,000 USD annually with certainty rather than being able to consume an infinite amount with a 99% probability and 35,991 USD annually with a 1% probability. If we instead draw on the results from the high payoff lottery in Holt and Laury, the results become less extreme, although only slightly. Indeed, as the fifth line shows, as many as 62% would prefer the safe option (36,000 USD annually) before a risky one with a 1% probability of being able to consume 35,942 USD per year and a 99% probability of gaining infinite consumption. If we consider the extreme case of 500% interest per year, the implied choices are still absurd. Moreover, as observed in the third and fourth column of Table 2, when considering CARA (instead of CRRA) preferences, the results are consistently even more extreme.[13]

3.2. More general HARA preferences[14]

The HARA instantaneous utility function, $u_t = (\alpha + \beta c_t)^{(\beta-1)/\beta}/(\beta - 1)$, implies decreasing absolute risk aversion ($\partial A_t/\partial c_t < 0$) for $\beta > 0$ and increasing absolute risk aversion for $\beta < 0$; we also observe decreasing relative risk aversion for $\alpha < 0$ and increasing relative risk aversion for $\alpha > 0$. It is also straightforward to see that this instantaneous utility function is globally concave (for parameter values such that it is defined, i.e. for $\alpha > -\beta c_t$) and that both A_t and R_t are everywhere decreasing in α and β. When an individual is indifferent between two lotteries, I and II, we have from (5) that

[13] An important reason for this is the pessimistic assumption regarding the subjects' future income that underlies the R estimates in Table 1.

[14] While HARA is the most commonly used flexible functional form, the second most commonly used is the so-called Expo-power utility function (Saha, 1993). Both of these flexible forms include CRRA and CARA as special cases. Johansson-Stenman (2009) also discusses some features of the Expo-power function.

$$\sum_{i=1}^{n} p_{\mathrm{I}}^{i}\big(\alpha+\beta\big(c^0+x_{\mathrm{I}}^{i}/S\big)\big)^{(\beta-1)/\beta}=\sum_{i=1}^{n} p_{\mathrm{II}}^{i}\big(\alpha+\beta\big(c^0+x_{\mathrm{II}}^{i}/S\big)\big)^{(\beta-1)/\beta}. \tag{15}$$

From (15) we can solve for β for a given value of α, and vice versa, or solve for either α or β for a specified relationship between them. It is convenient for presentational purposes to rewrite (15) for $\beta > 0$ as

$$\sum_{i=1}^{n} p_{\mathrm{I}}^{i}\big(\alpha/\beta+c^0+x_{\mathrm{I}}^{i}/S\big)^{(\beta-1)/\beta}=\sum_{i=1}^{n} p_{\mathrm{II}}^{i}\big(\alpha/\beta+c^0+x_{\mathrm{II}}^{i}/S\big)^{(\beta-1)/\beta}, \tag{16a}$$

whereas we for $\beta < 0$ instead have

$$\sum_{i=1}^{n} p_{\mathrm{I}}^{i}\big(-\alpha/\beta-c^0-x_{\mathrm{I}}^{i}/S\big)^{(\beta-1)/\beta}=\sum_{i=1}^{n} p_{\mathrm{II}}^{i}\big(-\alpha/\beta-c^0-x_{\mathrm{II}}^{i}/S\big)^{(\beta-1)/\beta}. \tag{16b}$$

Moreover, suppose now that α and β have been identified for an individual based on a risk experiment, such as the one by Holt and Laury. Let the same individual choose between a safe and a risky option regarding all future income levels, as in the previous case for CRRA and CARA preferences. Given indifference between the options, we can solve for c^L as follows:

$$c^L=\left(\frac{1}{1-p}\left(\frac{\alpha}{\beta}+c^S\right)^{(\beta-1)/\beta}-\frac{p}{1-p}\left(\frac{\alpha}{\beta}+c^H\right)^{(\beta-1)/\beta}\right)^{\beta/(\beta-1)}-\frac{\alpha}{\beta}. \tag{17}$$

Let us now again focus on the extreme case where the high income outcome implies an infinite consumption level. For $\beta < 1$, (17) then converges toward

$$c^L=(1-p)^{-\beta/(\beta-1)}\left(\frac{\alpha}{\beta}+c^S\right)-\frac{\alpha}{\beta}. \tag{18}$$

In Table 3 below, we calculate c^L for a very wide range of α/β.[15] As observed, the implied choices are still absurd for almost all values of α/β. Consider for example the case where $\alpha/\beta = -7000$. The number 35,977 in the fourth column should then be interpreted as follows: Assume that a student makes a choice between Option I and Option II in the first low-payoff lottery choice described in Table 1, and that he/she has HARA preferences where the relation between α and β is such that $\alpha = -7000\beta$, where β is a positive number. Based on the EUCT model with a 5% annual interest rate, this implies that if he/she chooses Option I, he/she would prefer a future income stream allowing him/her to for the rest of his/her life consume 36,000 USD annually with certainty rather than a risky alternative where he/she with a 99% probability would be able to consume an infinite amount and with a 1% probability would be able to consume 35,977 USD annually. This clearly seems implausible.

The only exception occurs where α/β is very close to the negative of the baseline income level, which in our case occurs where $c^0 = 10{,}000$. Indeed, when $\alpha/\beta = -10{,}000$, we can write utility of a lottery outcome at state i as

$$U^i=K^1\big(-10{,}000+c^0+X^i\big)^{1-0.146}=K^2\big(X^i\big)^{0.854},$$

where X^i represents the additional constant consumption level on top of c^0 and where K^1 and K^2 are constants. Hence, this function is equivalent to the CRRA EUI model at this value of c^0. This also means that the coefficient of the relative risk aversion would be the same as for the EUI case (reported in Table 1). Hence, we do not obtain the absurd choices in the example of future wages here. However, as shown below, we still obtain unreasonable large-stake choices close to the baseline consumption level.

So far we have drawn implications based on a single pairwise choice based on either the low-payoff or high-payoff lotteries of Holt and Laury. However, since we have two parameters in the HARA case, we can actually estimate the parameters consistent with being indifferent in the first low-payoff pairwise lottery choice *and* in the second high-payoff pairwise lottery choice. When doing this we obtain parameter values that are rather close to the case described above. Indeed, for the case where $r = 5\%$ annually, we can write utility as

$$U^i=K^3\big(-9999.84+c^0+X^i\big)^{1-0.479}=K^3\big(0.16+X^i\big)^{0.521},$$

where K^3 is a constant. Here too, there are no extreme risk averse choices with respect to the above thought experiment of future wages. The reason is that in order to match indifference in both the first low-payoff pairwise choice and the second high-payoff pairwise choice of Holt and Laury, the utility function has to have an extreme curvature in this region. This

[15] Note that for the instantaneous utility function to be defined, we must for $\beta > 0$ (i.e., where we have decreasing absolute risk aversion) have that $\alpha/\beta > -(c^0 + x^i/S)$ for all x^i. In the lottery about future wages we must then have that $\alpha/\beta > -c^L$. When $\beta < 0$ (i.e., where we have increasing absolute risk aversion), we must have that $\alpha/\beta < -(c^0 + x^i/S)$. This means that the instantaneous utility function in this range is not defined for a sufficiently large consumption level. In our future wage lottery we must then have that $\alpha/\beta < -c^H$. In order to illustrate this (rather unrealistic) range of the HARA utility function, we choose $c^H = 100{,}000$.

Table 3
Calculated implicit annual rest-of-life consumption in the unlucky outcome based on the EUCT model with HARA preferences, based on the experimental choices reported by Holt and Laury (2002), as reported by the first and the fifth pairwise choices in Table 1 in the present paper. The parameter $1/\beta$ is reported in brackets so that A and R can easily be calculated for different consumption levels.

α/β	Outcome in safe option USD/year	Lucky outcome (probability = 99%) USD/year	Unlucky outcome c^L (probability = 1%) based on low-payoff lottery choices; 66% would choose the safe option		Unlucky outcome c^L (probability = 1%) based on high-payoff lottery choices; 62% would choose the safe option	
			$R = 5\%$	$r = 500\%$	$r = 5\%$	$r = 500\%$
u characterized by CARA and IRRA						
$-\infty$	36,000	Infinite	35,999 [$-\infty$]	35,901 [$-\infty$]	35,993 [$-\infty$]	35,333 [$-\infty$]
u characterized by IARA and IRRA						
−500,000	36,000	100,000	35,998 [−943,122]	35,785 [−192,474]	35,985 [−139,010]	35,017 [−2174]
−200,000	36,000	100,000	35,998 [−365,700]	35,804 [−3850]	35,986 [−53,902]	35,147 [−887]
−110,000	36,000	100,000	35,998 [−192,474]	35,832 [−2026]	35,988 [−28,369]	35,228 [−443]
u is DARA and DRRA and chosen such that the choices based on the EUCT model with $c^0 = 10,000$ coincides with the choices based on the EUI model						
−10,000	36,000	36,200	17,619 [0.146]	17,619 [0.146]	19,496 [0.411]	19,496 [0.411]
u is DARA and DRRA and chosen consistent with the median subject in both the small-stake and the large-stake experiments of Holt and Laury, $r = 5\%$						
−9999.84	36,000	36,200	19,878 [0.479]	17,647 [0.151]	19,878 [0.479]	19,259 [0.374]
u is DARA and DRRA and chosen consistent with the median subject in both the small-stake and the large-stake experiments of Holt and Laury, $r = 500\%$						
−9964.41	36,000	36,200	34,610 [68.69]	21,825 [0.906]	30,937 [10.64]	21,825 [0.906]
u characterized by DARA and DRRA						
−9900	36,000	Infinite	35,380 [193]	10,489 [2.2]	3203 [29]	9900 [1.22]
−9000	36,000	Infinite	35,935 [1925]	30,308 [20]	35,565 [284]	18,187 [5.3]
−7000	36,000	Infinite	35,977 [5774]	33,857 [61]	35,843 [852]	27,438 [14.2]
−4000	36,000	Infinite	35,987 [11,549]	34,803 [122]	35,914 [1703]	30,892 [27]
u characterized by DARA and CRRA						
0	36,000	Infinite	35,991 [192,48]	35,188 [203]	35,942 [2838]	32,441 [45]
u characterized by DARA and IRRA						
5000	36,000	Infinite	35,993 [28871]	35,382 [304]	35,956 [4256]	33,254 [67]
20,000	36,000	Infinite	35,996 [57742]	35,577 [608]	35,970 [8511]	34,094 [134]
50,000	36,000	Infinite	35,997 [115485]	35,675 [1216]	35,977 [17,022]	34,525 [267]
200,000	36,000	Infinite	35,997 [404196]	35,745 [4255]	35,982 [59,577]	34,837 [933]
u characterized by CARA and IRRA						
∞	36,000	Infinite	35,999 [∞]	35,901 [∞]	35,993 [∞]	35,333 [∞]

Note: The instantaneous utility function u is not defined for $\alpha/\beta < -c^L$ for $\beta > 0$. For $\beta < 0$ u it is not defined for $\alpha/\beta > -c^H$.

in turn implies that the local risk aversion for small changes around $c^0 = 10{,}000$ will be extremely strong, whereas it will decrease rapidly for higher levels. For example, the relative risk aversion at the benchmark consumption level $c^0 = 10{,}000$ is given by

$$R = \frac{c/\beta}{\alpha/\beta + c} = \frac{10{,}000 \cdot 0.479}{-9999.84 + 10{,}000} = 29937.5,$$

whereas at the consumption level 36,000 we have

$$R = \frac{36{,}000 \cdot 0.479}{-9999.84 + 36{,}000} = 0.18.$$

Thus, we will obtain absurd large-stake risk aversion here too, but in another interval, namely close to the benchmark consumption level. Indeed, with these preferences, an individual would prefer a safe option with a future income corresponding to a constant consumption level of 10,000 per year, rather than a risky option where he/she with 99.99% probability would obtain an infinite amount and with a 0.01% probability would obtain an amount corresponding to 9999.84 per year (since utility converges to minus infinity at this level).

Consequently, we observe absurd large-stake implications based on the choice behavior in the Holt and Laury experiments for all HARA utility functions consistent with either the behavior in the small-stake experiment, the large-stake experiment, or both. Overall, it can then be concluded that the major conclusions about absurd large-stake implications based on the choice behavior in the Holt and Laury experiment hold much more generally than for CRRA and CARA preferences.

4. Discussion and conclusion

The explanatory power of EU theory has recently been discussed intensively (e.g., Rabin 2000a, 2000b; Rabin and Thaler, 2001; Cox and Sadiraj, 2006; Rubinstein, 2006). The present paper is concerned with the implications of this discussion for applied economics research dealing with risky choices. In such research, people often make decisions over time, where the standard model (here denoted the EUCT model) assumes that people maximize the expected present value of future instantaneous utility. The respective relations between the EUCT model and the EUW and EUI models are analyzed. It is shown that the EUCT model is equivalent to the EUW model when wealth is measured by the present value of all future consumption or income, whereas the EUCT model is equivalent to the EUI model only for CARA preferences.

However, as argued by Palacios-Huerta and Serrano (2006), the important question is not whether the EU model, or in our case the EUCT model, is literally correct. We know that it is not. What is important for applied economics is the extent to which the model can provide a reasonable approximation of actual behavior. Here the implications of the EUCT model have therefore been investigated based on data from a careful risk experiment reported in Holt and Laury (2002). The results suggest that the EUCT model is ill-suited to explain experimental behavior in such small- and intermediate-stake gambles. The calculated implicit risk-aversion parameters are found to be unreasonably large, and therefore cannot constitute concavity measures of universally valid instantaneous utility functions. For example, in the base case with CRRA preferences, making conservative or realistic assumptions regarding future wages, etc., the median coefficient of relative risk aversion is above 2000 even based on the high-stake lottery choices, despite the fact that most analysts seem to agree that R should be in the 0.5–3 range, or at least not be larger than 10.

Even more strikingly, the results suggest unreasonable implications in terms of what these degrees of risk aversion would correspond to for long-term risky choices in terms of the subjects' future consumption levels. Whether based on CRRA or CARA preferences, most subjects from Holt and Laury would in the base case EUCT model prefer a certain income enabling them to for the rest of their lifetime consume 36,000 USD annually instead of a risky alternative where they with a 1% probability would be able to consume 35,990 USD annually and with a 99% probability would be able to consume an infinite amount. Similar results are obtained based on more general HARA preferences. Moreover, since the results in Holt and Laury (2002) are in no way unique, but are in line with most small- and intermediate-stake experimental risk studies,[16] it can be concluded that the EUCT model appears inconsistent with observed experimental small- and intermediate-stake data. The same applies to at least some kinds of actual consumption behavior, such as additional insurances for electronic equipment. In the longer working-paper version of this paper (Johansson-Stenman, 2009), it is demonstrated that the main conclusions hold also under uncertain future incomes and for time-inconsistent preferences of present-bias type.

However, a caveat regarding the numerical results is in order. The numerical analysis here implicitly assumes that the choices reported in Holt and Laury are without errors, which is of course not the case in reality. Moreover, when extrapolating to implied choices at a much larger scale, as is done here, such errors will of course increase too, as pointed out by Cox and Harrison (2008). This means that the numerical results here should be interpreted with caution. Still, in order to escape the main conclusions, one would have to assume that the choices in Holt and Laury are almost solely random, which seems unlikely.

Consequently, we need another model to explain small- and intermediate-stake risk behavior, and there are several suggestions well worth considering in the literature; see, e.g., Rabin and Thaler (2001), Köbberling and Wakker (2005), Barberis et al. (2006), Cox et al. (2009), Heinemann (2008), and Rabin and Weizsäcker (forthcoming). Although it is beyond the scope of the present study to discriminate between these and other models, two remarks appear clear based on the findings here:

1. Loss aversion cannot explain the choice behavior in Holt and Laury, and hence not the numerical findings in this paper, since all outcomes were in the gain domain (unlike the thought experiments by Rabin and Thaler (2001)).
2. It appears that a successful model should include the element that a decision-maker who faces multiple decisions in each case tends to make a decision with insufficient regard to other decisions, and hence with insufficient regard to income or wealth from other sources.[17]

Finally, since repeated games also add payoffs over time, one may worry that repeated game theory is in trouble too. However, as noted by an advisory editor, the results from this paper do not justify such a conclusion. An important difference is that in repeated game theory, utility in a certain period depends only on the choices made in that period while in the EUCT model an increased consumption reduces future utility. The extent to which people integrate the payoffs from a certain stage in a game with the payoffs from other stages, as well as with background lifetime wealth, is a separate but important issue that is left for future research.

Acknowledgments

I am grateful for very constructive comments from two anonymous referees, an advisory editor, Martin Dufwenberg, Kjell Arne Brekke, Thomas Aronsson, Frank Heinemann, Robert Östling, Richard Thaler, Peter Wakker, Fredrik Carlsson, Matthias Sutter and seminar participants at Umeå University, as well as for financial support from the Swedish Research Council and the Swedish International Development Cooperation Agency (Sida).

[16] See, e.g., Cox and Sadiraj (2008) for a discussion and analysis of other recent risk experiments.

[17] This and similar phenomena have different names in the literature, including *decision isolation*, *mental accounting*, *narrow bracketing*, *narrow framing* and *partial asset integration*.

Appendix A. Proof of Proposition 1

We will first show that if u is a HARA instantaneous utility function as given by

$$u(c_t) = \frac{(\alpha + \beta c_t)^{(\beta-1)/\beta}}{\beta - 1}, \tag{A.1}$$

then i is implied. By differentiating (A.1) with respect to c_t and using (3) we obtain

$$c_t = \frac{\lambda^{-\beta} e^{-\beta(\rho - r)t} - \alpha}{\beta}. \tag{A.2}$$

From the budget condition (2) we get

$$\lambda^{-\beta} = \frac{\beta(r(1-\beta) + \beta\rho)}{(1 - e^{-(r(1-\beta)+\beta\rho)T})}\left(Y + \frac{\alpha}{\beta r}\left(1 - e^{-rT}\right)\right),$$

which substituted into (A.2) gives the optimal consumption path as

$$c_t^* = M e^{\beta(r-\rho)t}\left(Y + \frac{\alpha}{\beta}\frac{1 - e^{-rT}}{r}\right) - \frac{\alpha}{\beta},$$

where

$$M \equiv \frac{r(1-\beta) + \beta\rho}{1 - e^{-(r(1-\beta)+\beta\rho)T}}$$

is a constant. Hence, c_t^* can be written as an affine function of Y as follows:

$$c_t^* = a_t + b_t Y, \tag{A.3}$$

where

$$a_t = M\frac{\alpha}{\beta} e^{\beta(r-\rho)t}\frac{1 - e^{-rT}}{r} - \frac{\alpha}{\beta}$$

and

$$b_t = M e^{\beta(r-\rho)t}$$

are independent of Y. By substituting (A.3) into (A.1), we obtain:

$$u(c_t) = M^{(\beta-1)/\beta}\frac{(S\alpha + \beta Y)^{(\beta-1)/\beta}}{\beta - 1} e^{(\beta-1)(r-\rho)t}, \tag{A.4}$$

where S is the annuity factor. Substituting (A.4) into (1), finally, and some straightforward simplifications, give

$$U = V(Y) = M^{(\beta-1)/\beta}\frac{(S\alpha + \beta Y)^{(\beta-1)/\beta}}{\beta - 1}\int_0^T e^{(\beta-1)(r-\rho)t} e^{-\rho t}\, dt = K\frac{(S\alpha + \beta Y)^{(\beta-1)/\beta}}{\beta - 1}, \tag{A.5}$$

where $K = M^{-1/\beta}$ is a constant. Hence, we have shown that when u is HARA as described by (A.1), then (A.5), and hence i, is implied. In order to complete the proof we will finally show that (A.5) implies ii. This follows directly from substituting $Y = Sc^0$ into (A.5), so that

$$U = K\frac{(S\alpha + \beta Sc^0)^{(\beta-1)/\beta}}{\beta - 1} = K'\frac{(\alpha + \beta c^0)^{(\beta-1)/\beta}}{\beta - 1}, \tag{A.6}$$

where $K' = KS^{(\beta-1)/\beta}$ is a constant. □

References

Attanasio, O.P., Browning, M., 1995. Consumption over the life cycle and over the business cycle. Amer. Econ. Rev. 85, 1118–1137.

Barberis, N., Huang, M., Thaler, R.H., 2006. Individual preferences, monetary gambles, and stock market participation: A case for narrow framing. Amer. Econ. Rev. 96, 1069–1090.

Blundell, R., Browning, M., Meghir, C., 1994. Consumer demand and the life-cycle allocation of household expenditures. Rev. Econ. Stud. 61, 57–80.

Cox, J.C., Harrison, G.W., 2008. Risk aversion in experiments: An introduction. Chapter 1. In: Cox, J.C., Harrison, G.W. (Eds.), Risk Aversion in Experiments. In: Research in Experimental Economics, vol. 12. Emerald JAI Press, Cornwall, pp. 1–7.

Cox, J.C., Sadiraj, V., 2006. Small- and large-stakes risk aversion: Implications of concavity calibration for decision theory. Games Econ. Behav. 56, 45–60.

Cox, J.C., Sadiraj, V., 2008. Risky decisions in the large and in the small: Theory and experiment. Chapter 2. In: Cox, J.C., Harrison, G.W. (Eds.), Risk Aversion in Experiments. In: Research in Experimental Economics, vol. 12. Emerald JAI Press, Cornwall, pp. 9–40.
Cox, J.C., Sadiraj, V., Vogt, B., Dasgupta, U., 2009. Is there a plausible theory for risky decisions? Updated version of Experimental Economics Center Working Paper 2008-04, Georgia State University.
Deaton, A., 1992. Understanding Consumption. Oxford University Press, Oxford.
Gollier, C., 2001. The Economics of Risk and Time. MIT Press, Boston.
Gomes, F., Michaelides, A., 2005. Optimal life-cycle asset allocation: Understanding the empirical evidence. J. Finance 60, 869–904.
Hall, R.E., 1978. Stochastic implications of the life cycle/permanent income hypothesis; theory and evidence. J. Polit. Economy 86, 971–987.
Harrison, G.W., Lau, M.I., Rutström, E.E., 2007. Estimating risk attitudes in Denmark: A field experiment. Scand. J. Econ. 109, 341–368.
Heinemann, F., 2008. Measuring risk aversion and the wealth effect. In: Cox, J.C., Harrison, G.W. (Eds.), Risk Aversion in Experiments. In: Research in Experimental Economics, vol. 12. Emerald JAI Press, Cornwall, pp. 293–313.
Holt, C.A., Laury, S.K., 2002. Risk aversion and incentive effects. Amer. Econ. Rev. 92, 1644–1655.
Johansson-Stenman, O., 2009. Risk aversion and expected utility of consumption over time. Working Paper no. 351, Department of Economics, University of Gothenburg.
Kocherlakota, N., 1996. The equity premium: It's still a puzzle. J. Econ. Lit. 34, 42–71.
Köbberling, V., Wakker, P.P., 2005. An index of loss aversion. J. Econ. Theory 122, 119–131.
Merton, R.C., 1971. Optimum consumption and portfolio rules in a continuous time model. J. Econ. Theory 3, 373–413.
Nordhaus, W., 2007. A review of the Stern review on the economics of climate change. J. Econ. Lit. 45, 686–702.
Palacios-Huerta, I., Serrano, R., 2006. Rejecting small gambles under expected utility. Econ. Letters 91, 250–259.
Rabin, M., 2000a. Risk aversion and expected utility theory: A calibration theorem. Econometrica 68, 1281–1292.
Rabin, M., 2000b. Diminishing marginal utility of wealth cannot explain risk aversion. In: Kahneman, D., Tversky, A. (Eds.), Choices, Values and Frames. Cambridge University Press.
Rabin, M., Thaler, R.H., 2001. Anomalies: Risk aversion. J. Econ. Perspect. 15, 219–232.
Rabin, M., Thaler, R.H., 2002. Response: (Anomalies: Risk aversion). J. Econ. Perspect. 16, 229–230.
Rabin, M., Weizsäcker, G., forthcoming. Narrow bracketing and dominated choices. Amer. Econ. Rev.
Rubinstein, A., 2006. Dilemmas of an economic theorist. Econometrica 74, 865–884.
Saha, A., 1993. Expo-power utility: A flexible form for absolute and relative risk aversion. Amer. J. Agr. Econ. 75, 905–913.
Samuelson, L., 2005. Foundations of human sociality: A review essay. J. Econ. Lit. 43, 488–497.
Schechter, L., 2007. Risk aversion and expected-utility theory: A calibration exercise. J. Risk Uncertainty 35, 67–76.
Stern, N., 2007. The Economics of Climate Change: The Stern Review. Cambridge University Press, Cambridge, UK.
Vissing-Jørgensen, A., 2002. Limited asset market participation and the elasticity of intertemporal substitution. J. Polit. Economy 4, 825–853.